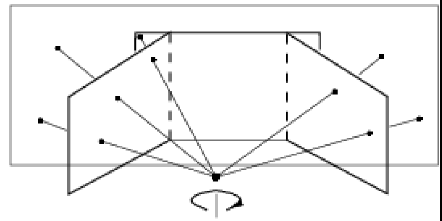


Image warping and stitching

Thurs Oct 15

Last time

- Feature-based alignment
  - 2D transformations
  - Affine fit
  - RANSAC

Robust feature-based alignment

- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T* (small group of putative matches that are related by *T*)
  - *Verify* transformation (search for other matches consistent with *T*)

Source: L. Lazebnik

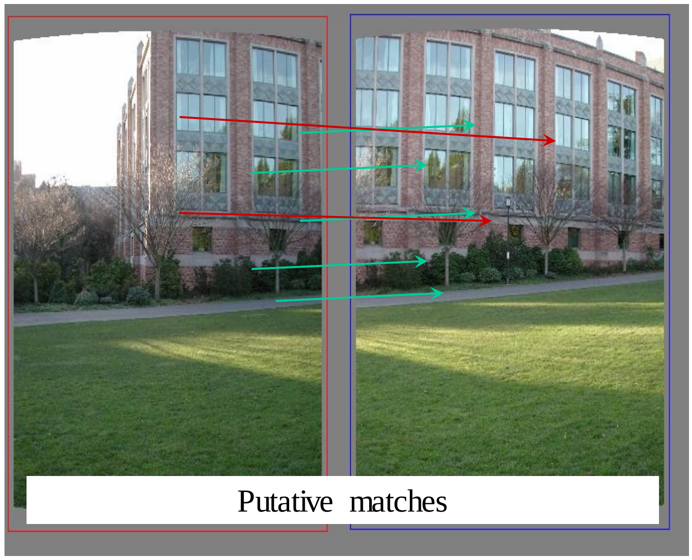
RANSAC: General form

RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

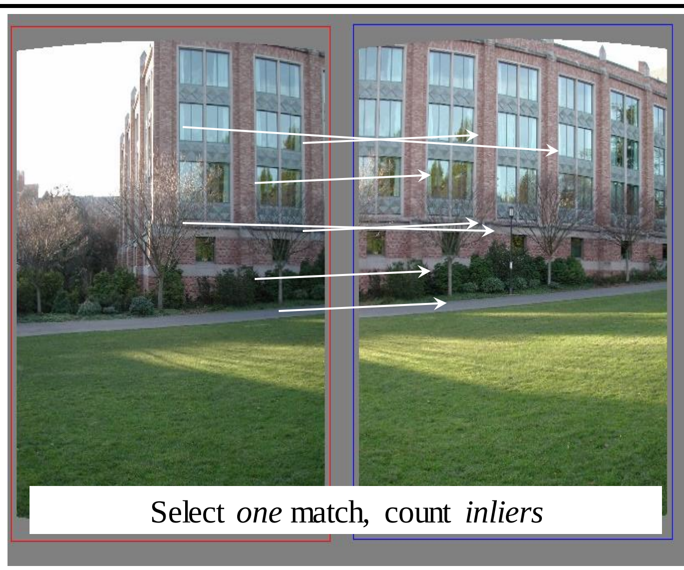
RANSAC example: Translation



Putative matches

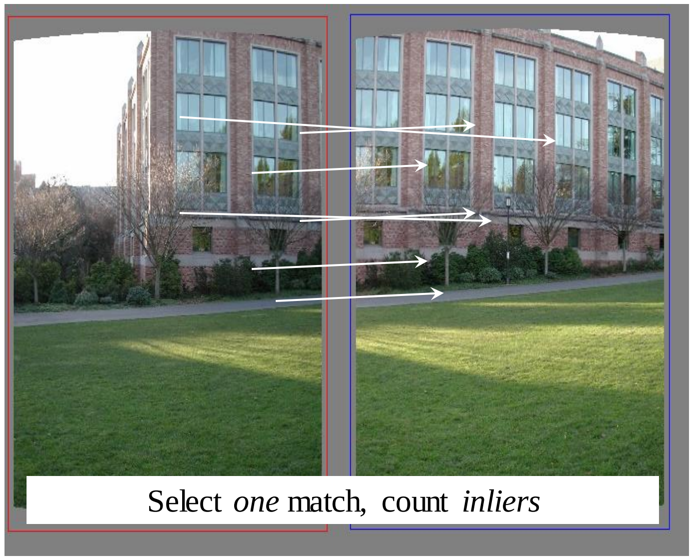
Source: Rick Szeliski

RANSAC example: Translation

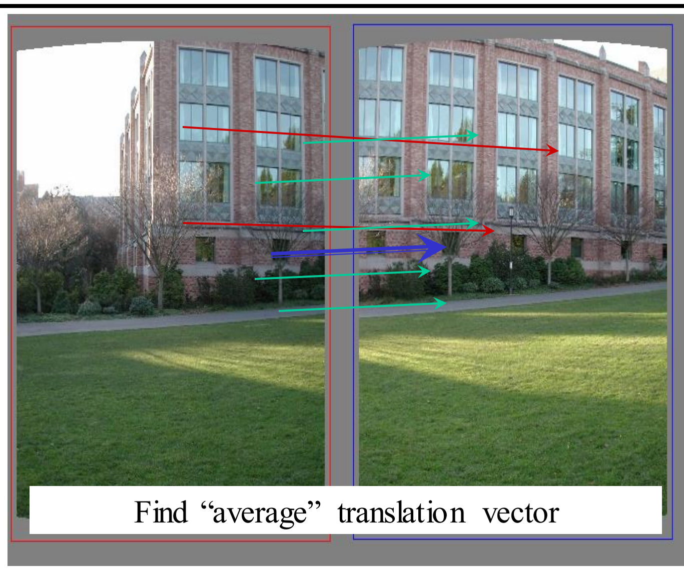


Select *one* match, count *inliers*

RANSAC example: Translation

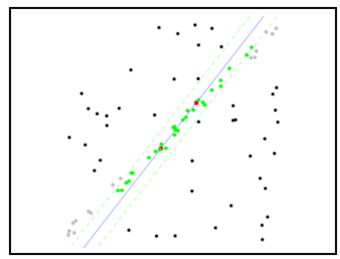


RANSAC example: Translation



RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Lots of parameters to tune
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)
  - Can't always get a good initialization of the model based on the minimum number of samples



Lana Lazebnik

Another example

Automatic scanned document rotater using Hough lines and RANSAC

Gen Hough vs RANSAC

GHT
- Single correspondence -> vote for all consistent parameters
- Represents uncertainty in the model parameter space
- Linear complexity in number of correspondences and number of voting cells; beyond 4D vote space impractical
- Can handle high outlier ratio

RANSAC
- Minimal subset of correspondences to estimate model -> count inliers
- Represents uncertainty in image space
- Must search all data points to check for inliers each iteration
- Scales better to high-d parameter spaces

Kristen Grauman

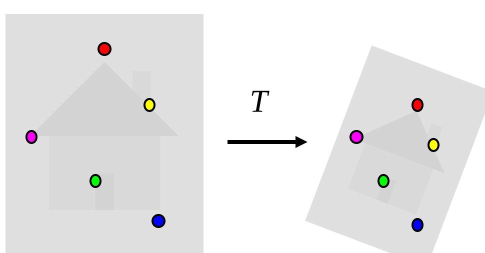
Today

- Image mosaics
  - Fitting a 2D transformation
    - Affine, Homography
  - 2D image warping
  - Computing an image mosaic

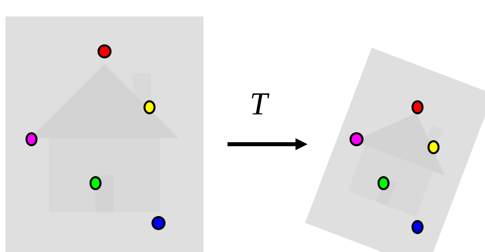
Mosaics

Obtain a wider angle view by combining multiple images.

Main questions



Alignment: Given two images, what is the transformation between them?



Warping: Given a source image and a transformation, what does the transformed output look like?

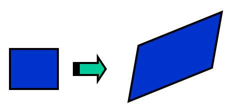
2D Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Parallel lines remain parallel



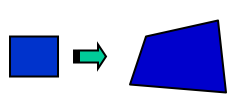
Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective transformations:

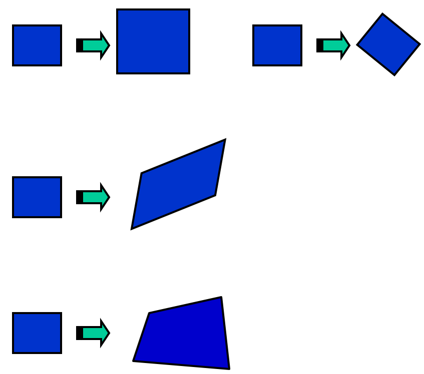
- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel



2D transformation models

- Similarity (translation, scale, rotation)
- Affine
- Projective (homography)



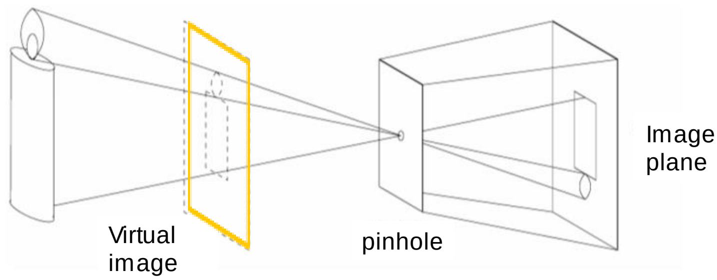
How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - (If there are more images, repeat)
- …but **wait**, why should this work at all?
  - What about the 3D geometry of the scene?
  - Why aren't we using it?

Source: Steve Seitz

Pinhole camera

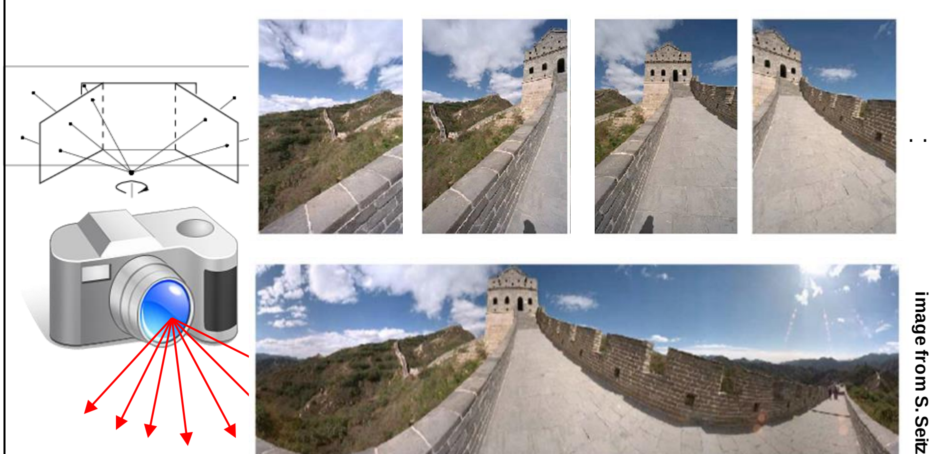
- Pinhole camera is a simple model to approximate imaging process, perspective **projection**.



If we treat pinhole as a point, only one ray from any given point can enter the camera.

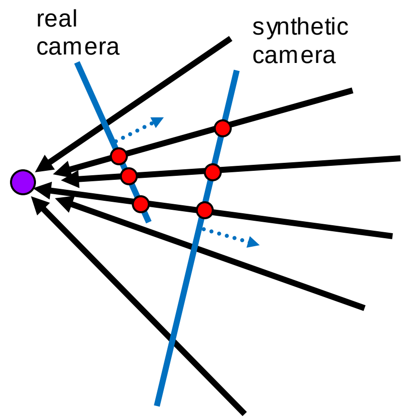
Fig from Forsyth and Ponce

Mosaics



Obtain a wider angle view by combining multiple images.

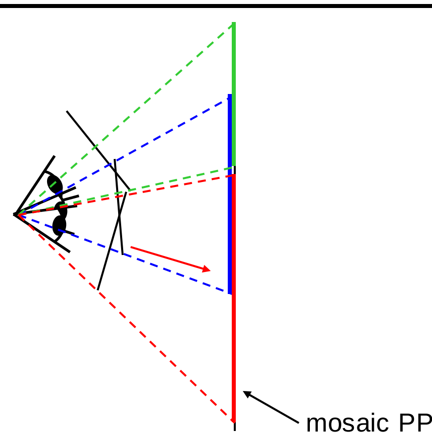
Mosaics: generating synthetic views



Can generate any synthetic camera view
as long as it has **the same center of projection**!

Source: Alyosha Efros

Image reprojection



The mosaic has a natural interpretation in 3D

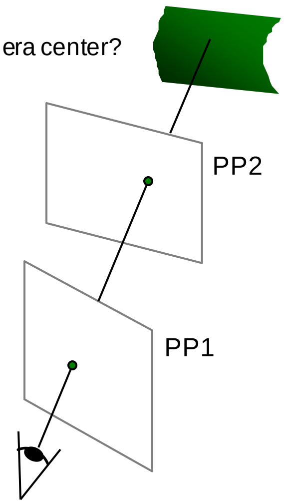
- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*

Source: Steve Seitz

Image reprojection

Basic question

- How to relate two images from the same camera center?
  - how to map a pixel from PP1 to PP2



Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D **image warp** from one image to another.

Source: Alyosha Efros

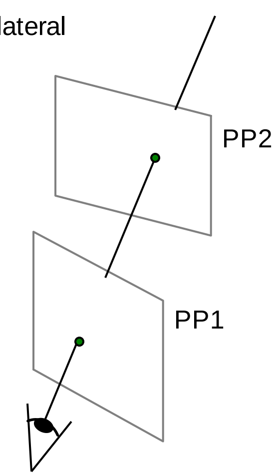
Image reprojection: Homography

A projective transform is a mapping between any two PPs with the same center of projection

- rectangle should map to arbitrary quadrilateral
- parallel lines aren't
- but must preserve straight lines

called **Homography**

$$\underset{\mathbf{p'}}{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}} = \underset{\mathbf{H}}{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}} \underset{\mathbf{p}}{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$



Source: Alyosha Efros

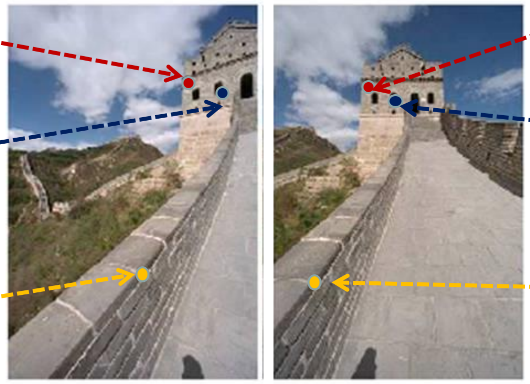
Homography

(x_1, y_1) (x'_1, y'_1)

(x_2, y_2) (x'_2, y'_2)

$\vdots$

(x_n, y_n) (x'_n, y'_n)



To **compute** the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of **H** are the unknowns…

Solving for homographies

p' = Hp

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Can set scale factor i=1. So, there are 8 unknowns.

Set up a system of linear equations:

Ah = b

where vector of unknowns h = [a,b,c,d,e,f,g,h]^T

Need at least 8 eqs, but the more the better…

Solve for h. If overconstrained, solve using least-squares:

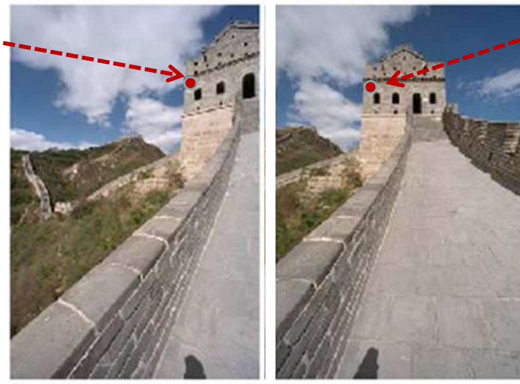
$$\min \|Ah - b\|^2$$

```
>> help lmdivide
```

BOARD

Homography

(x, y)



$\left(\frac{wx'}{w}, \frac{wy'}{w} \right) = (x', y')$

To **apply** a given homography **H**

- Compute **p'** = **Hp** (regular matrix multiply)
- Convert **p'** from homogeneous to image coordinates

$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{\mathbf{p'}} = \underbrace{\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\mathbf{p}}$$

RANSAC for estimating homography

RANSAC loop:

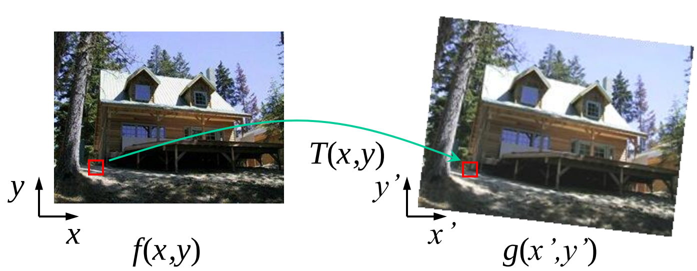
1. Select four feature pairs (at random)
2. Compute homography H
3. Compute *inliers* where $SSD(p_i', \boldsymbol{H}p_i) < \varepsilon$
4. Keep largest set of inliers
5. Re-compute least-squares H estimate on all of the inliers

Slide credit: Steve Seitz

Today

- Image mosaics
  - Fitting a 2D transformation
    - Affine, Homography
  - 2D image warping
  - Computing an image mosaic

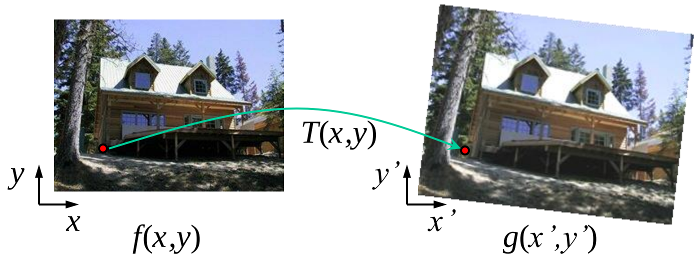
Image warping



Given a coordinate transform and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

Slide from Alyosha Efros, CMU

Forward warping



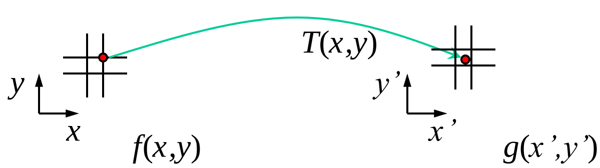
Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

Slide from Alyosha Efros, CMU

Forward warping



Send each pixel $f(x,y)$ to its corresponding location

$(x',y') = T(x,y)$ in the second image

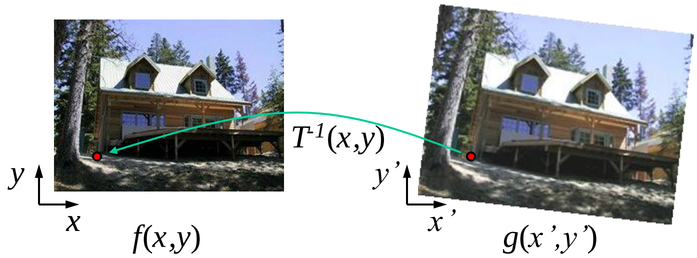
Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

– Known as "splatting"

Slide from Alyosha Efros, CMU

Inverse warping



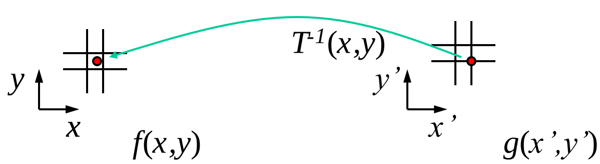
Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

Slide from Alyosha Efros, CMU

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

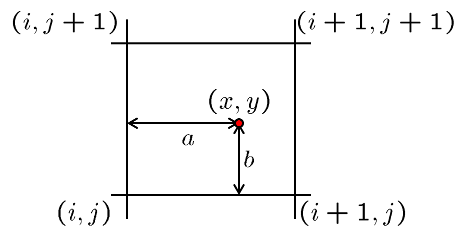
– nearest neighbor, bilinear…

`>> help interp2`

Slide from Alyosha Efros, CMU

Bilinear interpolation

Sampling at *f(x,y):*



$$
\begin{aligned}
f(x,y) = \; & (1-a)(1-b) && f[i,j] \\
& +a(1-b) && f[i+1,j] \\
& +ab && f[i+1,j+1] \\
& +(1-a)b && f[i,j+1]
\end{aligned}
$$

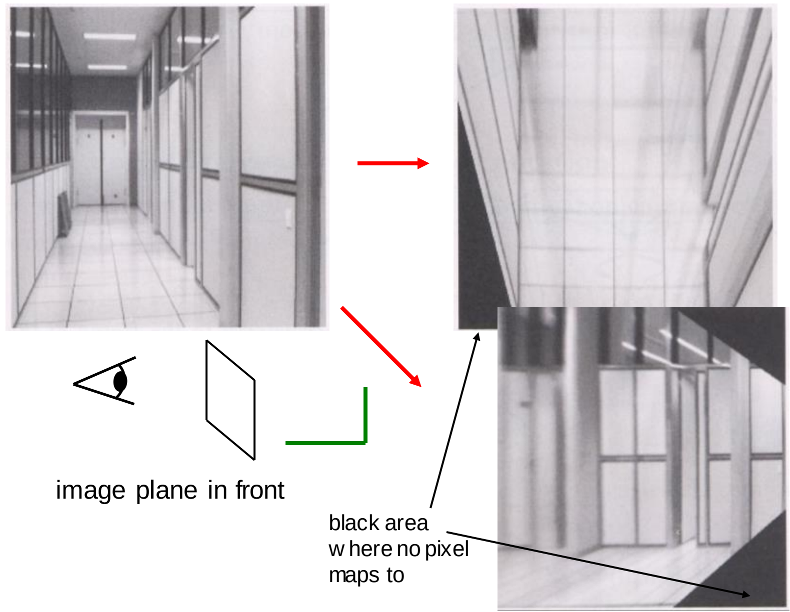
Slide from Alyosha Efros, CMU

Recap: How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation (homography) between second image and first using corresponding points.
  - Transform the second image to overlap with the first.
  - Blend the two together to create a mosaic.
  - (If there are more images, repeat)

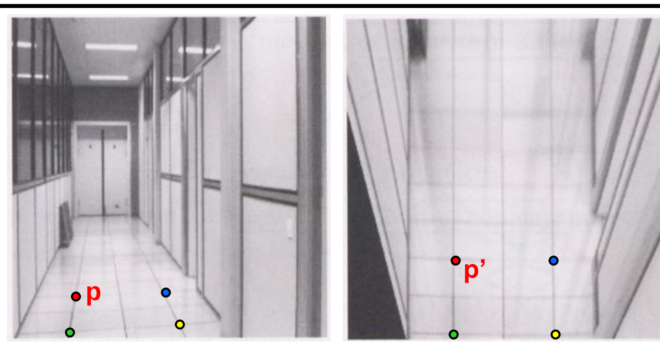
Source: Steve Seitz

Image warping with homographies



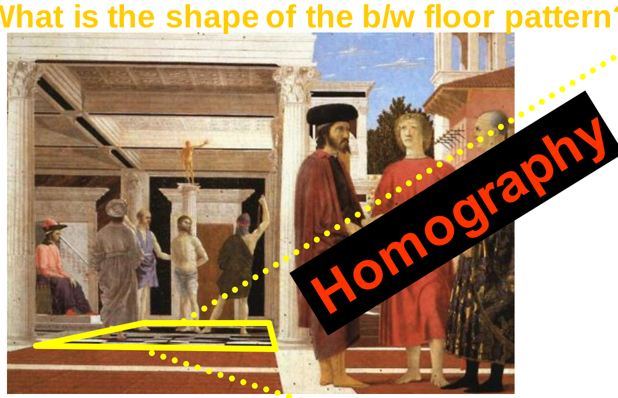
Source: Steve Seitz

Image rectification



Analysing patterns and shapes

What is the shape of the b/w floor pattern?



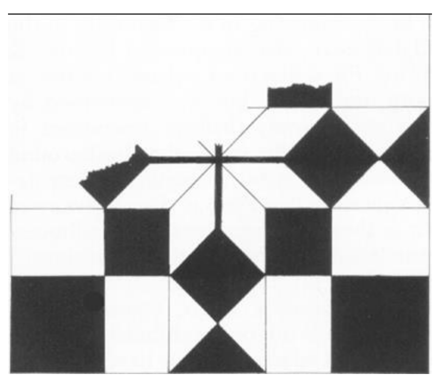
The floor (enlarged)

Automatically rectified floor

Slide from Antonio Criminisi

Analysing patterns and shapes

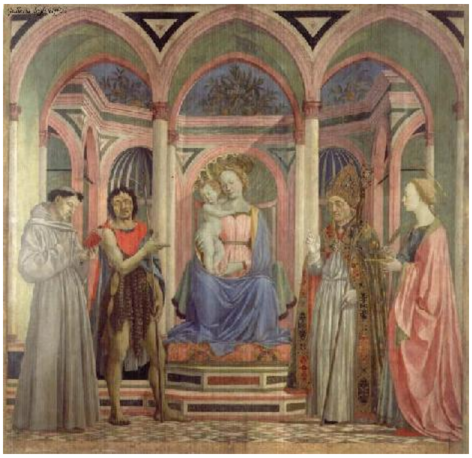
Automatic rectification



From Martin Kemp *The Science of Art* *(manual reconstruction)*

Slide from Antonio Criminisi

Analysing patterns and shapes



What is the (complicated) shape of the floor pattern?

Automatically rectified floor

***St. Lucy Altarpiece,* D. Veneziano**

Slide from Criminisi

Analysing patterns and shapes

Automatic rectification

From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

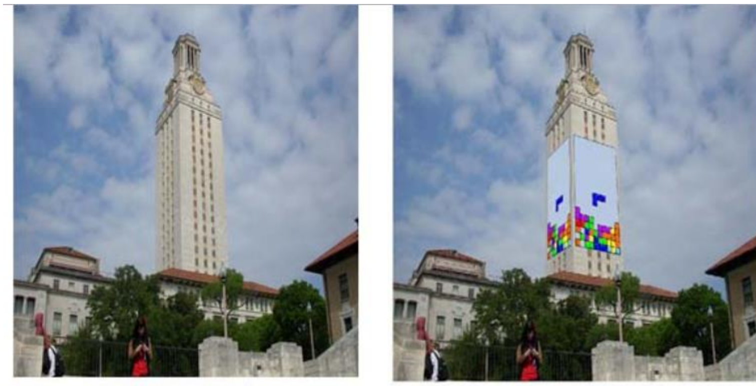
Andrew Harp

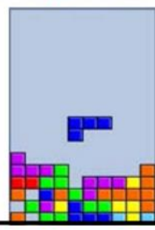
Andy Luong

Sung Ju Hwang

Ekapol Chuangsuwanich, CMU

Jesse Vera





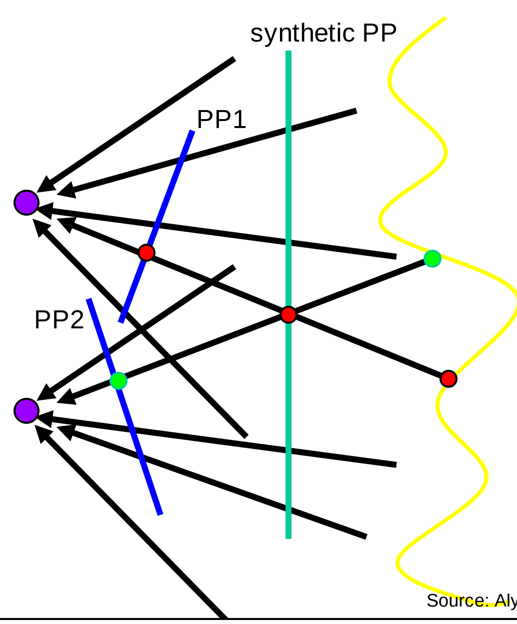
Kevin Gladstone

HP frames commercials

- http://www.youtube.com/watch?v=2RPl5vPEoQk

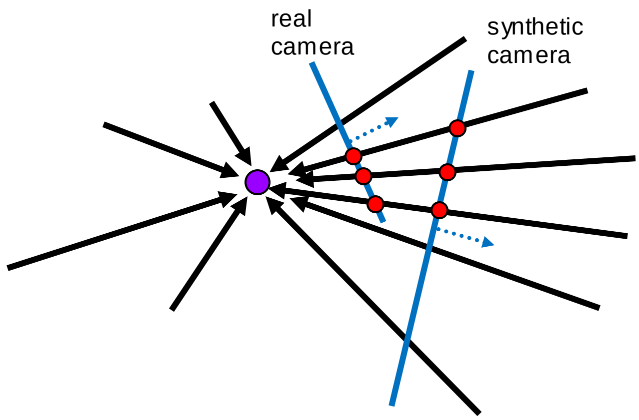
Changing camera center

Does it still work?



Source: Alyosha Efros

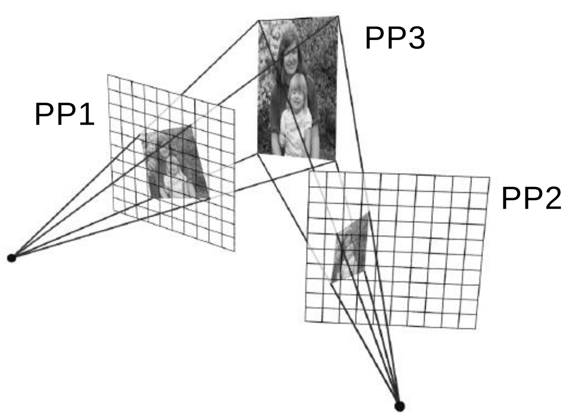
Recall: same camera center



Can generate synthetic camera view
as long as it has **the same center of projection**.

Source: Alyosha Efros

…Or: Planar scene (or far away)



PP3 is a projection plane of both centers of projection,
so we are OK!

This is how big aerial photographs are made

Source: Alyosha Efros

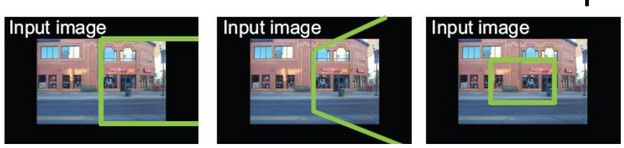
Boundary extension

- Wide-Angle Memories of Close-Up Scenes, Helene Intraub and Michael Richardson, Journal of Experimental Psychology: Learning, Memory, and Cognition, 1989, Vol. 15, No. 2, 179-187

Creating and Exploring a Large Photorealistic Virtual Space

Josef Sivic, Biliana Kaneva, Antonio Torralba, Shai Avidan and William T. Freeman, Internet Vision Workshop, CVPR 2008.
http://www.youtube.com/watch?v=E0rboU10rPo

Creating and Exploring a Large Photorealistic Virtual Space



Current view, and desired view in green

Synthesized view from new camera

Induced camera motion

Summary: alignment & warping

- Write **2d transformations** as matrix-vector multiplication (including translation when we use homogeneous coordinates)
- Perform **image warping** (forward, inverse)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.

Panoramas: main steps

- **1. Collect correspondences (manually for now)**
- **2. Solve for homography matrix H**
  - Least squares solution
- **3. Warp content from one image frame to the other to combine: say im1 into im2 reference frame**
- **4. Overlay im2 content onto the warped im1 content.**

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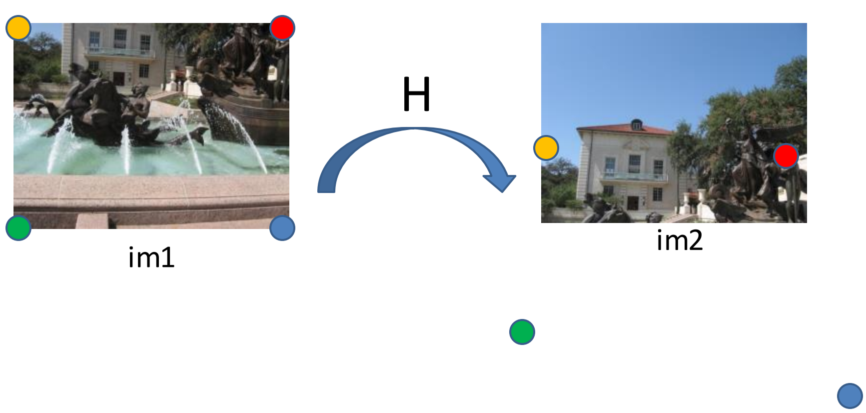
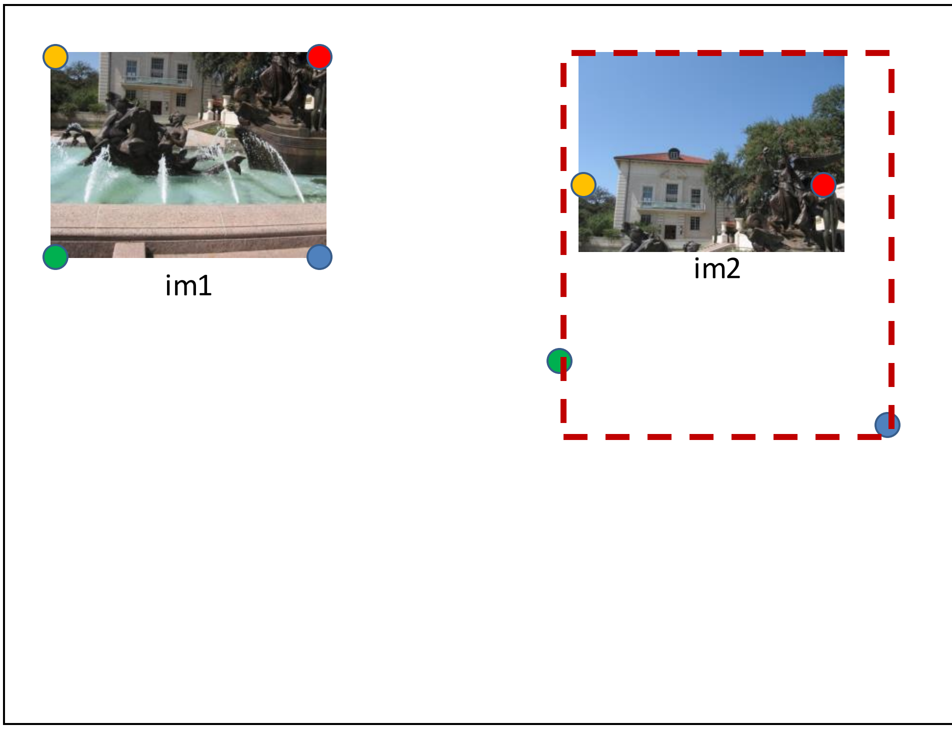
  - Determine bounds of the new combined image:
    - Where will the corners of im1 fall in im2's coordinate frame?
    - We will attempt to lookup colors for any of these positions we can get from im1.
- **4. Overlay im2 content onto the warped im1 content.**



(Assuming we have solved for the H that maps points from im1 to im2.)

$$\begin{bmatrix} wx_2 \\ wy_2 \\ w \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$



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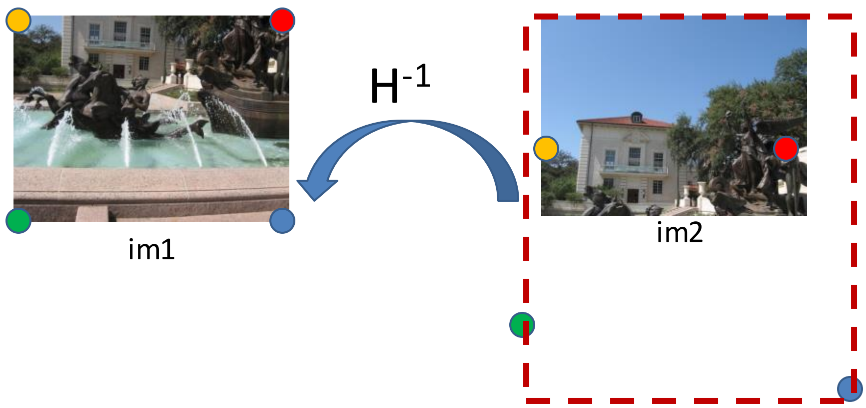
    - Where will the corners of im1 fall in im2's coordinate frame?
    - We will attempt to lookup colors for any of these positions we can get from im1.
  - Inverse warp:
    - Compute coordinates in im1's reference frame (via homography) for all points in that range.
    - Lookup all colors for all these positions from im1 (interp2)
- **4. Overlay im2 content onto the warped im1 content.**



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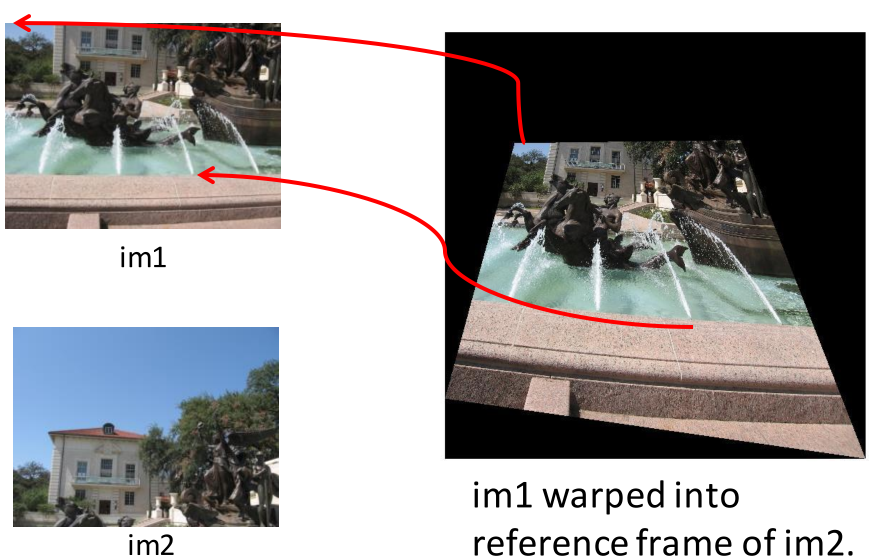
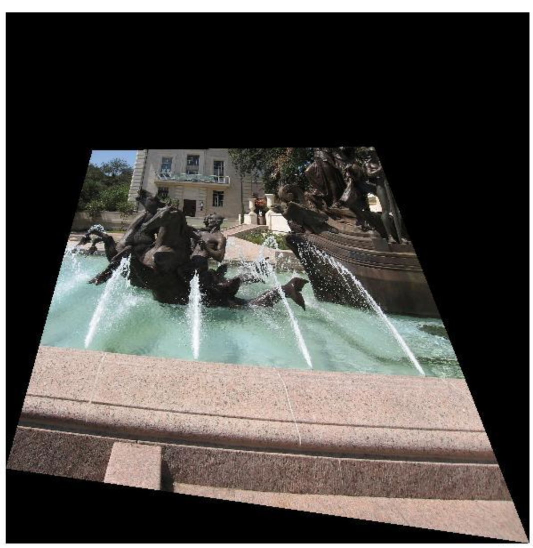


im1 warped into reference frame of im2.

Use interp2 to ask for the colors (possibly interpolated) from im1 at all the positions needed in im2's reference frame.

Panoramas: main steps

- **1. Collect correspondences (manually for now)**
- **2. Solve for homography matrix H**
  - Least squares solution
- **3. Warp content from one image frame to the other to combine: say im1 into im2 reference frame**
  - Determine bounds of the new combined image:
    - Where will the corners of im1 fall in im2's coordinate frame?
    - We will attempt to lookup colors for any of these positions we can get from im1.
  - Inverse warp:
    - Compute coordinates in im1's reference frame (via homography) for all points in that range.
    - Lookup all colors for all these positions from im1 (interp2)
- **4. Overlay im2 content onto the warped im1 content.**
  - Careful about new bounds of the output image



Summary: alignment & warping

- Write **2d transformations** as matrix-vector multiplication (including translation when we use homogeneous coordinates)
- Perform **image warping** (forward, inverse)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.