

Last time: How to stitch a panorama?

- Basic Procedure
 - Take a sequence of images from the same position
 - Rotate the camera about its optical center
 - Compute transformation (homography) between second image and first using corresponding points.
 - Transform the second image to overlap with the first.
 - Blend the two together to create a mosaic.
 - (If there are more images, repeat)

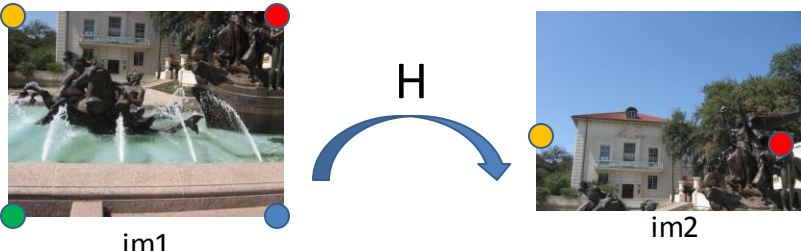
Source: Steve Seitz

Panoramas: main steps

- 1. Collect correspondences (manually for now)
- 2. Solve for homography matrix H
 - Least squares solution
- 3. Warp content from one image frame to the other to combine:
say im1 into im2 reference frame
- 4. Overlay im2 content onto the warped im1 content.

Panoramas: main steps

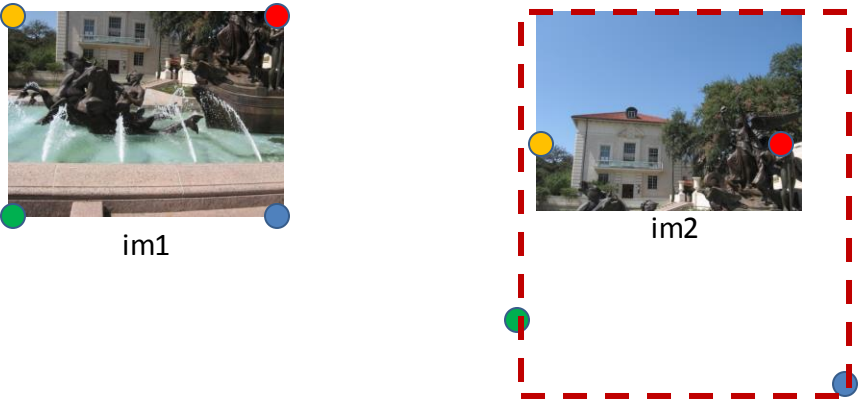
- 1. Collect correspondences (manually for now)
- 2. Solve for homography matrix H
 - Least squares solution
- 3. Warp content from one image frame to the other to combine:
say im1 into im2 reference frame
 - Determine bounds of the new combined image:
 - Where will the corners of im1 fall in im2's coordinate frame?
 - We will attempt to lookup colors for any of these positions we can get from im1.
- 4. Overlay im2 content onto the warped im1 content.



im1

im2

(Assuming we have solved for the H that maps points from im1 to im2.)

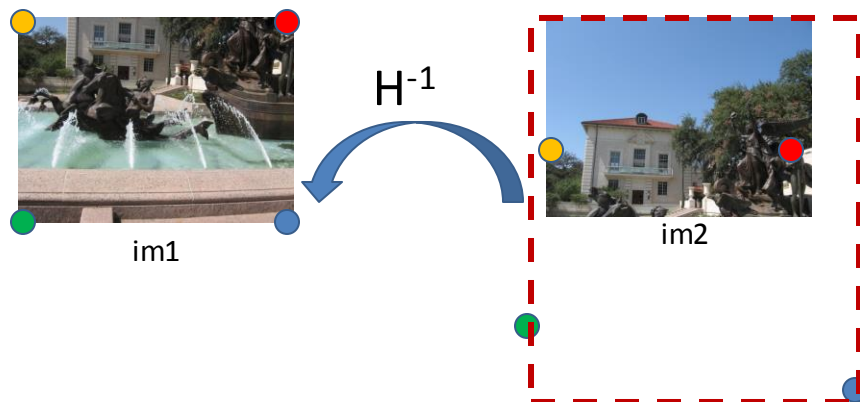
$$\begin{bmatrix} wx_2 \\ wy_2 \\ w \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$


im1

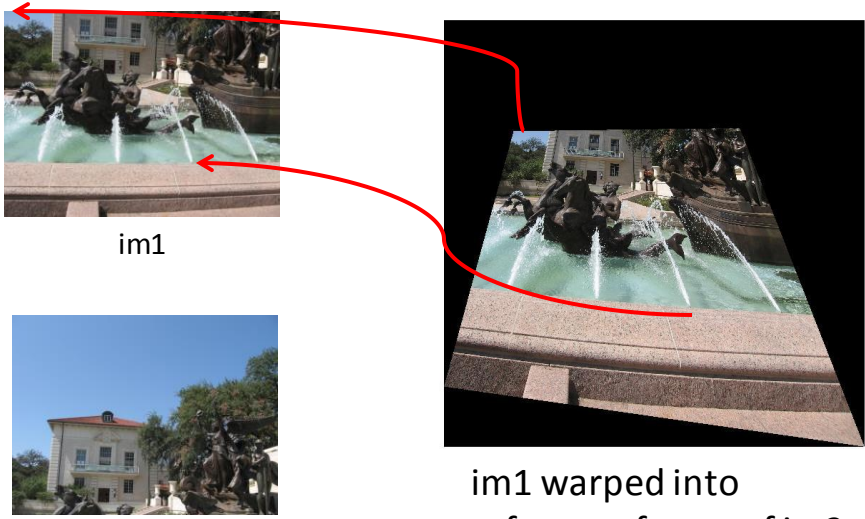
im2

Panoramas: main steps

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 - Determine bounds of the new combined image:
 - Where will the corners of im1 fall in im2's coordinate frame?
 - We will attempt to lookup colors for any of these positions we can get from im1.
 - Inverse warp:
 - Compute coordinates in im1's reference frame (via homography) for all points in that range.
 - Lookup all colors for all these positions from im1 (interp2)
- **4. Overlay im2 content onto the warped im1 content.**



(Assuming we have solved for the H that maps points from *im1* to *im2*.)



im1

im2

im1 warped into reference frame of im2.

Use interp2 to ask for the colors (possibly interpolated) from im1 at all the positions needed in im2's reference frame.

Panoramas: main steps

- **1. Collect correspondences (manually for now)**
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 - Compute coordinates in im1's reference frame (via homography) for all points in that range.
 - Lookup all colors for all these positions from im1 (interp2)
- **4. Overlay im2 content onto the warped im1 content.**
 - Careful about new bounds of the output image

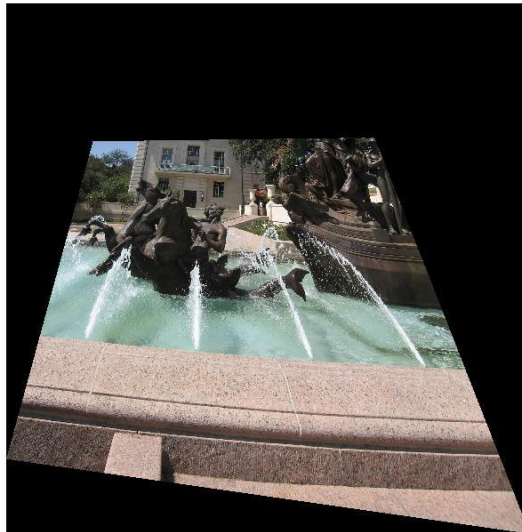
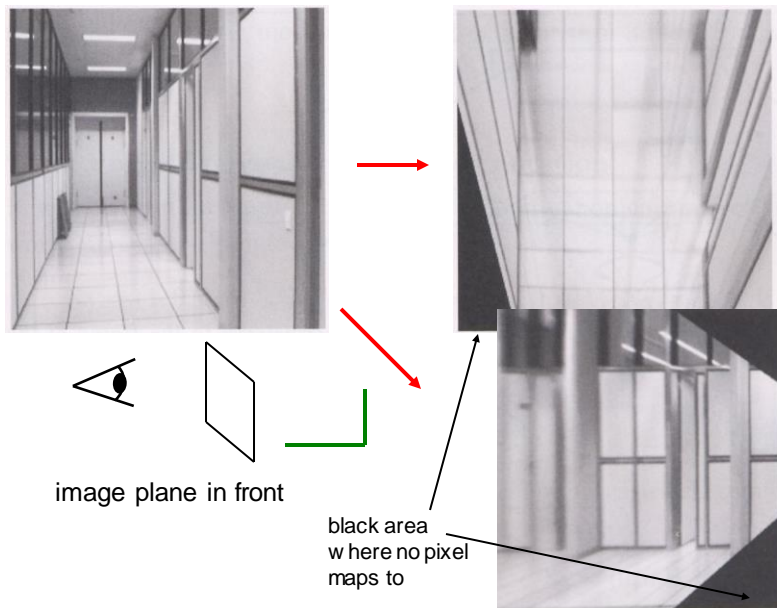
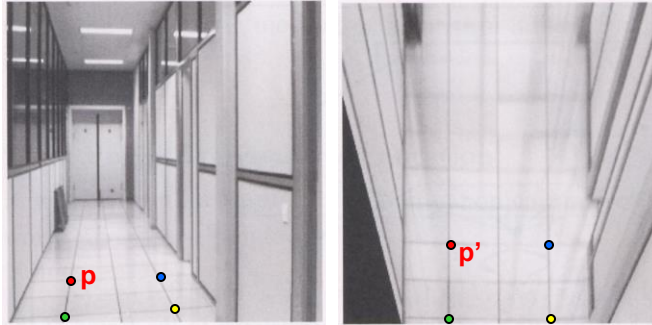


Image warping with homographies



Source: Steve Seitz

Image rectification



Analysing patterns and shapes

What is the shape of the b/w floor pattern?



The floor (enlarged)

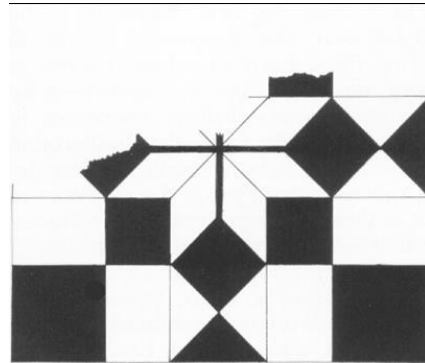


Automatically
rectified floor

Slide from Antonio Criminisi

Analysing patterns and shapes

Automatic rectification



From Martin Kemp *The Science of Art*
(*manual reconstruction*)

Slide from Antonio Criminisi

Analysing patterns and shapes



What is the (complicated)
shape of the floor pattern?



Automatically rectified floor

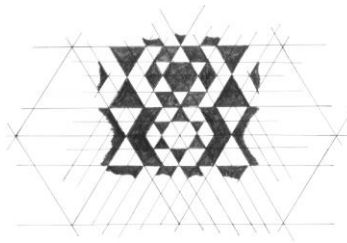
St. Lucy Altarpiece, D. Veneziano

Slide from Criminisi

Analysing patterns and shapes



**Automatic
rectification**



**From Martin Kemp, *The Science of Art*
(manual reconstruction)**

Slide from Criminisi



Andrew Harp



Andy Luong



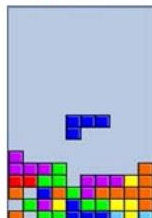
Sung Ju Hwang



Ekapol Chuangsuwanich, CMU



Jesse Vera

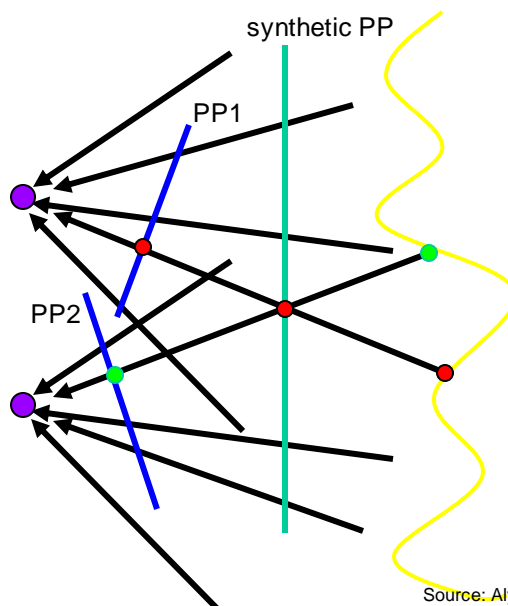


Kevin Gladstone



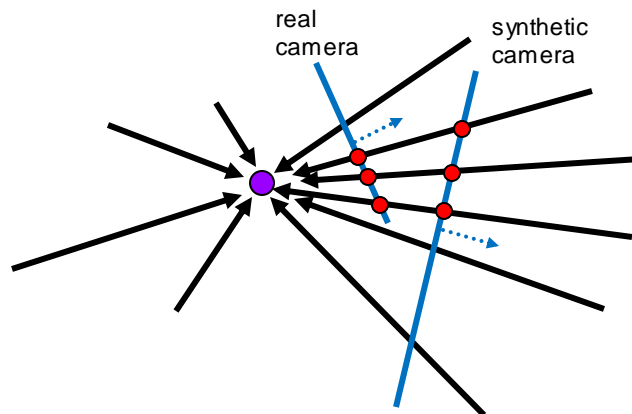
Changing camera center

Does it still work?



Source: Alyosha Efros

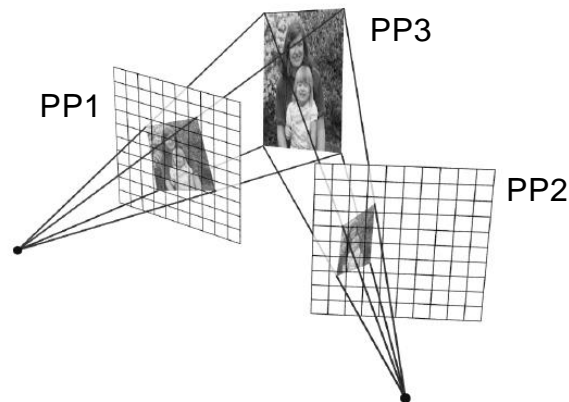
Recall: same camera center



Can generate synthetic camera view
as long as it has **the same center of projection**.

Source: Alyosha Efros

...Or: Planar scene (or far away)



PP3 is a projection plane of both centers of projection,
so we are OK!

This is how big aerial photographs are made

Source: Alyosha Efros



Boundary extension

- Wide-Angle Memories of Close-Up Scenes, Helene Intraub and Michael Richardson, *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 1989, Vol. 15, No. 2, 179-187

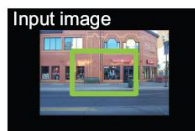
Creating and Exploring a Large Photorealistic Virtual Space



Josef Sivic, Biliana Kaneva, Antonio Torralba, Shai Avidan and William T. Freeman, Internet Vision Workshop, CVPR 2008.

<http://www.youtube.com/watch?v=E0rboU10rPo>

Creating and Exploring a Large Photorealistic Virtual Space



Current view, and
desired view in green

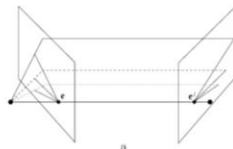
Synthesized view from
new camera

Induced camera
motion

Summary: alignment & warping

- Write **2d transformations** as matrix-vector multiplication (including translation when we use homogeneous coordinates)
- Perform **image warping** (forward, inverse)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.

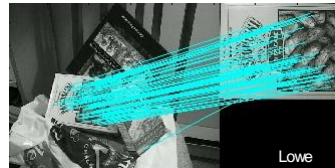
Multiple views



Hartley and Zisserman



Multi-view geometry,
matching, invariant
features, stereo vision



Why multiple views?

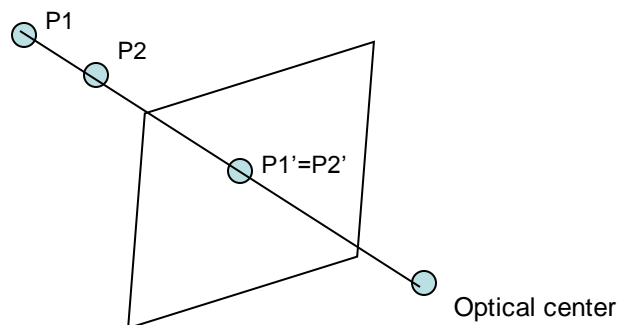
- Structure and depth are inherently ambiguous from single views.



Images from Lana Lazebni

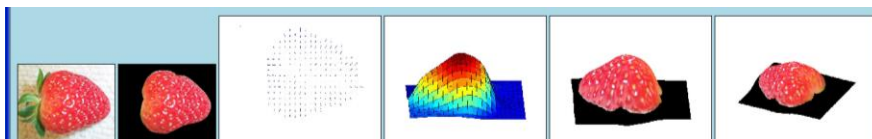
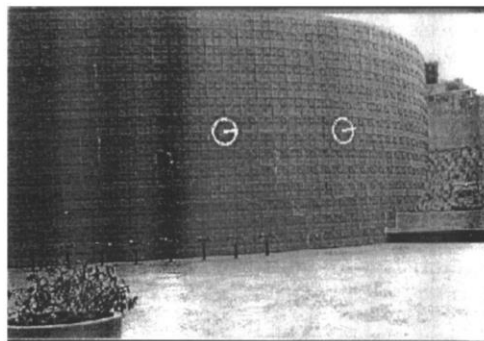
Why multiple views?

- Structure and depth are inherently ambiguous from single views.



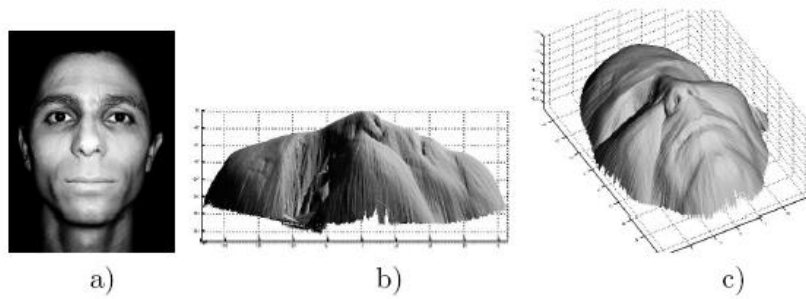
- What cues help us to perceive 3d shape and depth?

Texture



[From [A.M. Loh, The recovery of 3-D structure using visual texture patterns](#), PhD thesis]

Shading



[Figure from Prados & Faugeras 2006]

Perspective effects



Image credit: S. Seitz

Motion



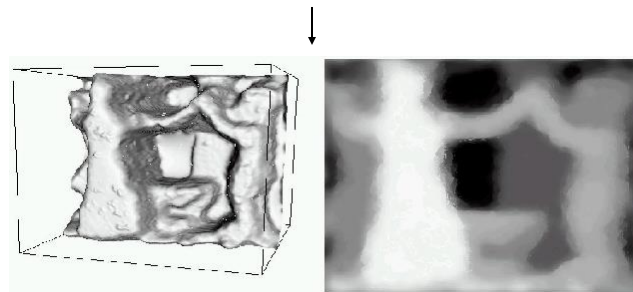
Figures from L. Zhang

<http://www.brainconnection.com/teasers/?main=illusion/motion-shape>

Focus/defocus



Images from
same point of
view, different
camera
parameters



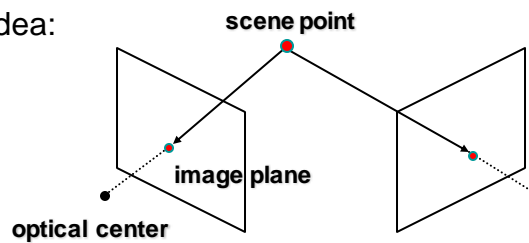
3d shape / depth
estimates

[figs f from H. Jin and P. Favaro, 2002]

Estimating scene shape

- “Shape from X”: Shading, Texture, Focus, Motion...
- **Stereo:**
 - shape from “motion” between two views
 - infer 3d shape of scene from two (multiple) images from different viewpoints

Main idea:

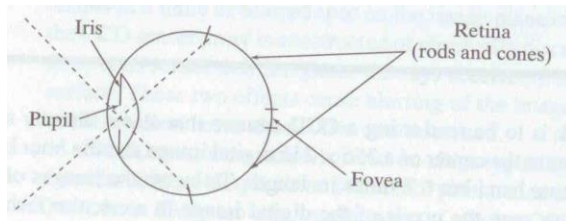


Outline

- Human stereopsis
- Epipolar geometry and the epipolar constraint
 - Case example with parallel optical axes
 - General case with calibrated cameras
- Stereo solutions
 - Correspondences
 - Additional constraints

Human eye

Rough analogy with human visual system:



Pupil/Iris – control amount of light passing through lens

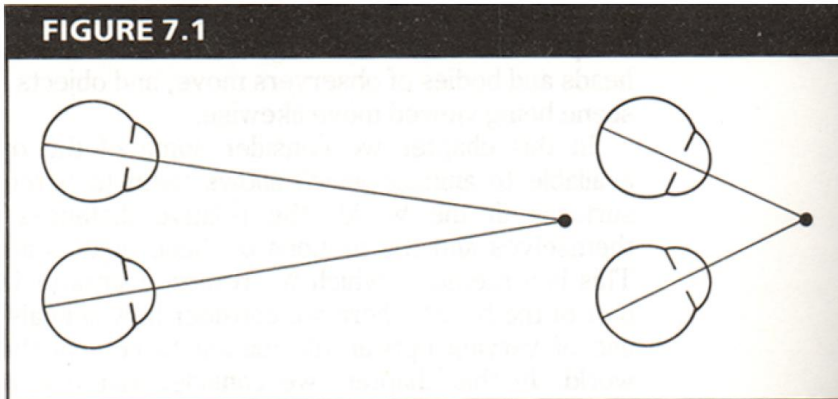
Retina - contains sensor cells, where image is formed

Fovea – highest concentration of cones

Fig from Shapiro and Stockman

Human stereopsis: disparity

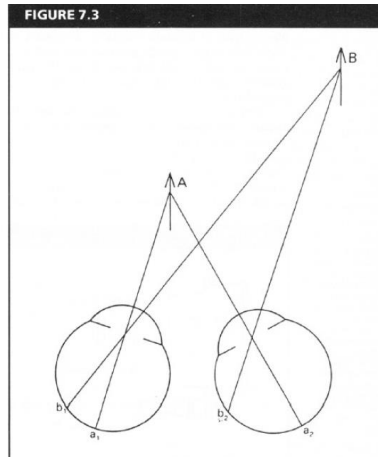
FIGURE 7.1



From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Human eyes **fixate** on point in space – rotate so that corresponding images form in centers of fovea.

Human stereopsis: disparity

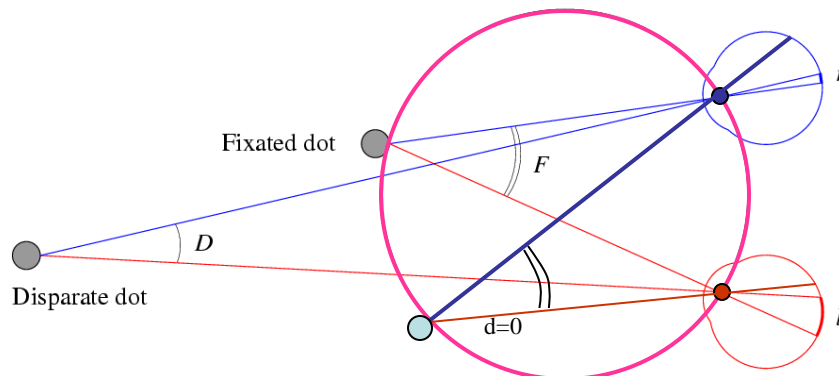


From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Adapted from David Forsyth, UC Berkeley

Disparity occurs when eyes fixate on one object; others appear at different visual angles

Human stereopsis: disparity



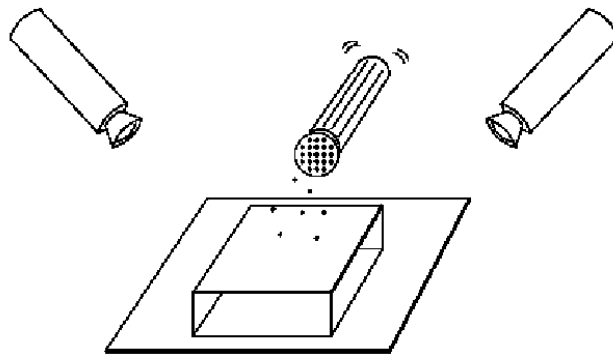
Disparity: $d = r - l = D - F$

Forsyth & Ponce

Random dot stereograms

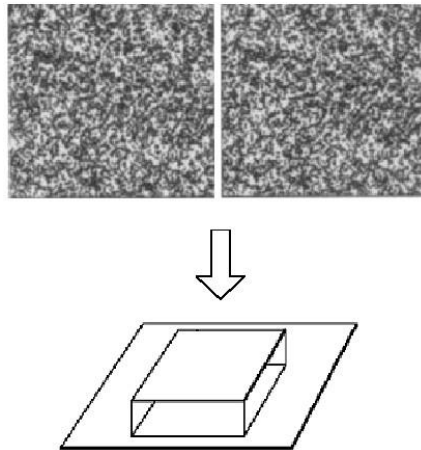
- Julesz 1960: Do we identify local brightness patterns before fusion (monocular process) or after (binocular)?
- To test: pair of synthetic images obtained by randomly spraying black dots on white objects

Random dot stereograms



Forsyth & Ponce

Random dot stereograms



Random dot stereograms

- When viewed monocularly, they appear random; when viewed stereoscopically, see 3d structure.
- Conclusion: human binocular fusion not directly associated with the physical retinas; must involve the central nervous system
- Imaginary “*cyclopean retina*” that combines the left and right image stimuli as a single unit

Stereo photography and stereo viewers

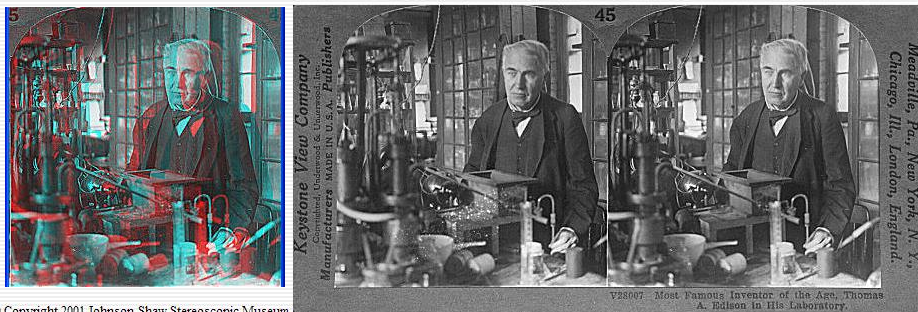
Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.



Invented by Sir Charles Wheatstone, 1838



Image from fisher-price.com



© Copyright 2001 Johnson-Shaw Stereoscopic Museum

<http://www.johnsonshawmuseum.org>



© Copyright 2001 Johnson-Shaw Stereoscopic Museum

<http://www.johnsonshawmuseum.org>



Public Library Stereoscopic Looking Room, Chicago, by Phillips, 1923





http://www.well.com/~jimg/stereo/stereo_list.html

Autostereograms



Exploit disparity as depth cue using single image.

(Single image random dot stereogram, Single image stereogram)

Images from magiceye.com

Autostereograms



Images from magiceye.com

Outline

- Human stereopsis
- Stereograms
- Epipolar geometry and the epipolar constraint
 - Case example with parallel optical axes
 - General case with calibrated cameras

Stereo vision



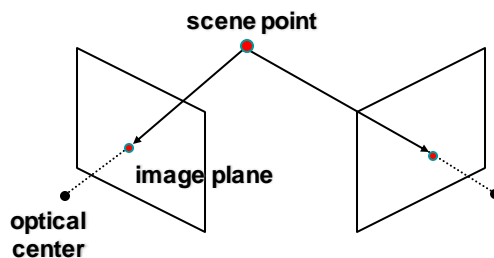
Two cameras, simultaneous views



Single moving camera and static scene

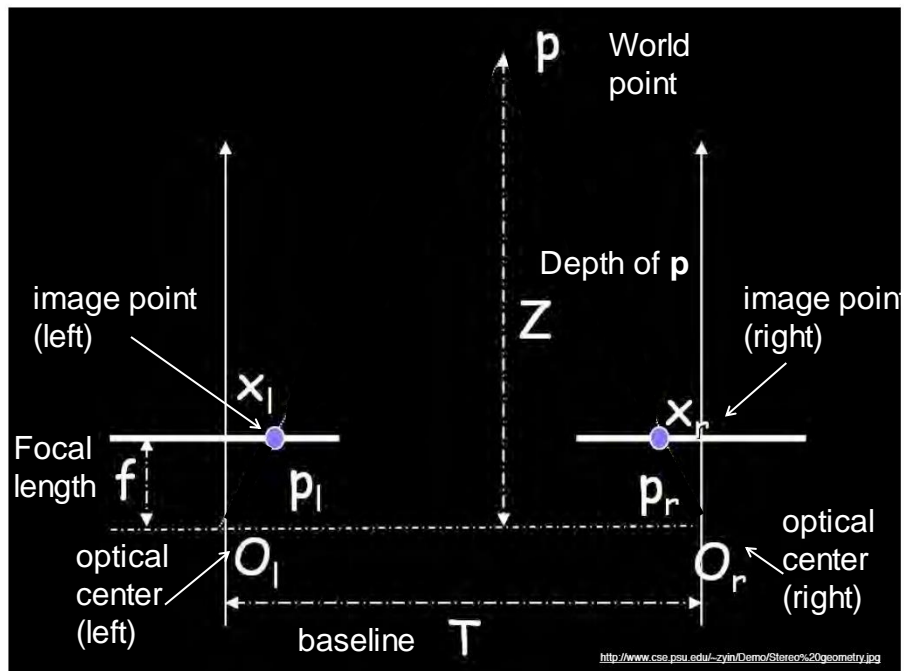
Estimating depth with stereo

- **Stereo**: shape from “motion” between two views
- We’ll need to consider:
 - Info on camera pose (“calibration”)
 - Image point correspondences



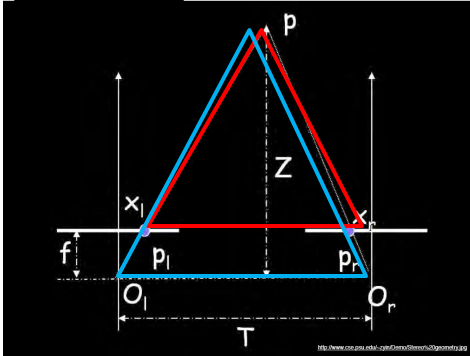
Geometry for a simple stereo system

- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):



Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles (p_l, P, p_r) and (O_l, P, O_r) :

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_r - x_l}$$

disparity \rightarrow $x_r - x_l$

Depth from disparity

image $I(x,y)$



Disparity map $D(x,y)$

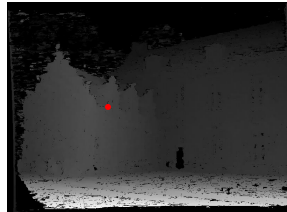


image $I'(x',y')$



$$(x', y') = (x + D(x, y), y)$$

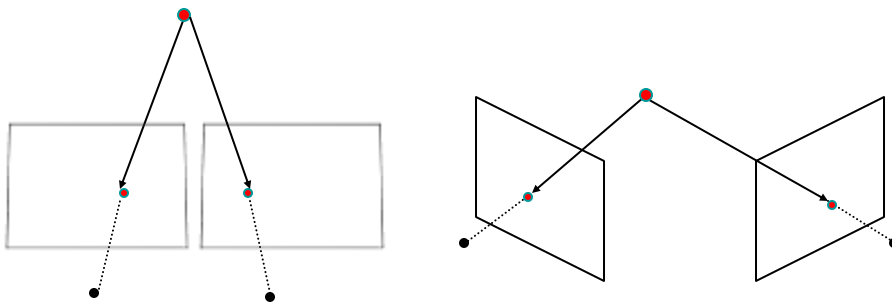
So if we could find the **corresponding points** in two images, we could **estimate relative depth**...

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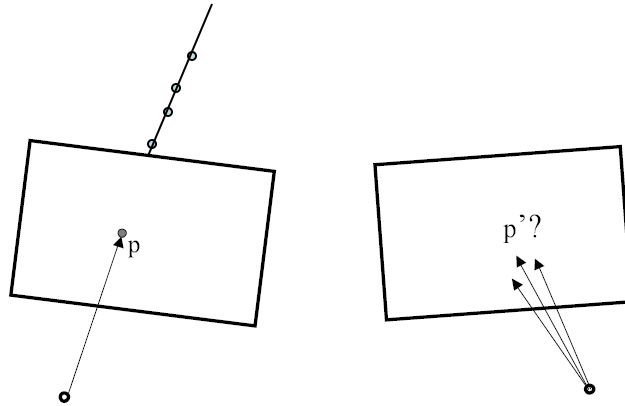
General case, with calibrated cameras

- The two cameras need not have parallel optical axes.



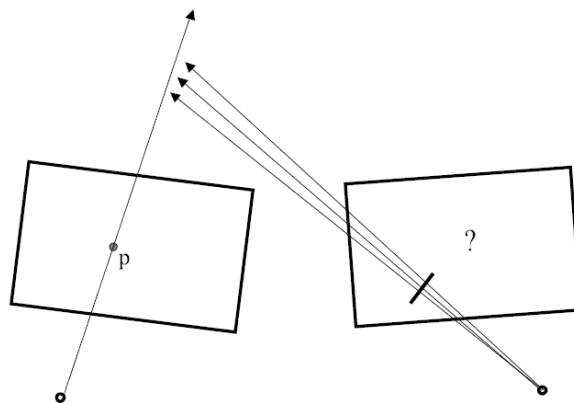
Vs.

Stereo correspondence constraints

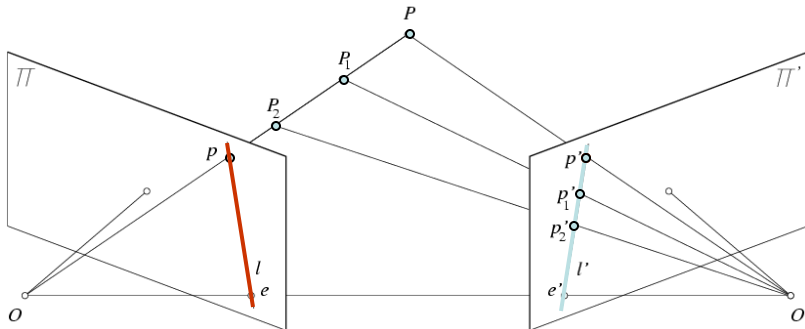


- Given p in left image, where can corresponding point p' be?

Stereo correspondence constraints



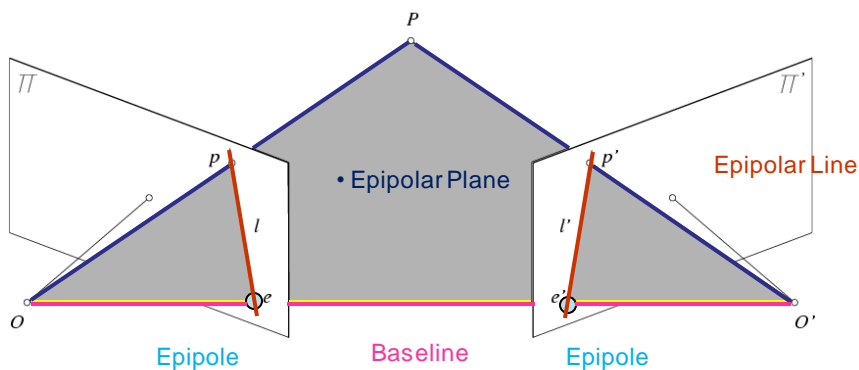
Epipolar constraint



Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view.

- It must be on the line carved out by a plane connecting the world point and optical centers.

Epipolar geometry



<http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html>

Epipolar geometry: terms

- **Baseline:** line joining the camera centers
 - **Epipole:** point of intersection of baseline with image plane
 - **Epipolar plane:** plane containing baseline and world point
 - **Epipolar line:** intersection of epipolar plane with the image plane
-
- All epipolar lines intersect at the epipole
 - An epipolar plane intersects the left and right image planes in epipolar lines

Why is the epipolar constraint useful?

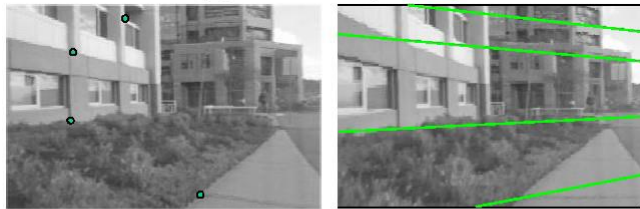
Epipolar constraint



This is useful because it reduces the correspondence problem to a 1D search along an epipolar line.

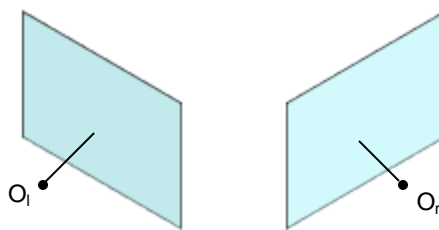
Image from Andrew Zisserman

Example

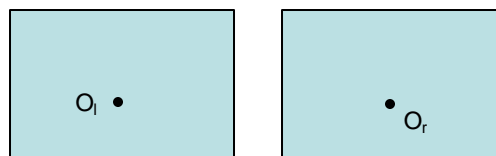


What do the epipolar lines look like?

1.



2.



Example: converging cameras

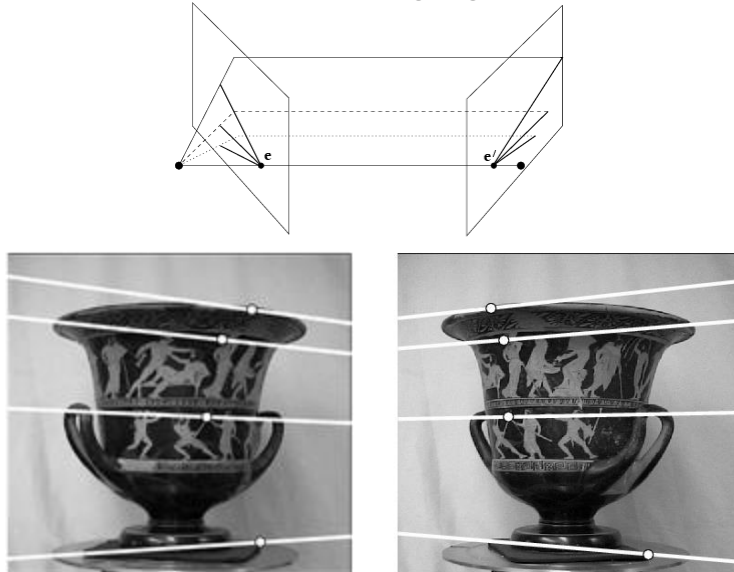


Figure from Hartley & Zisserman

Example: parallel cameras

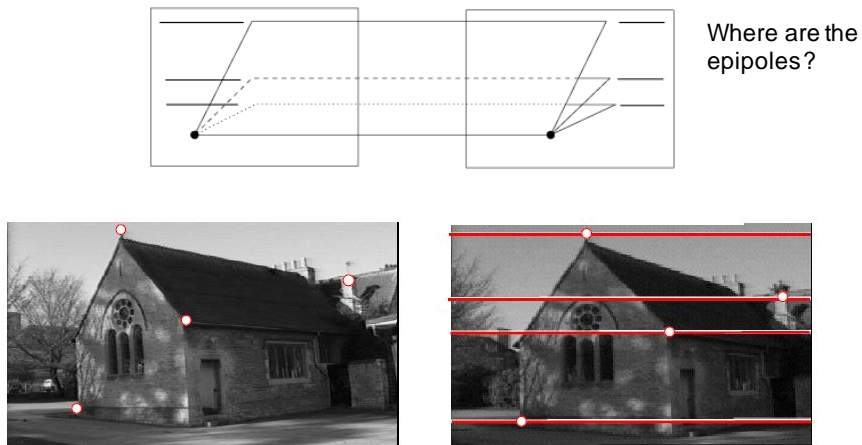
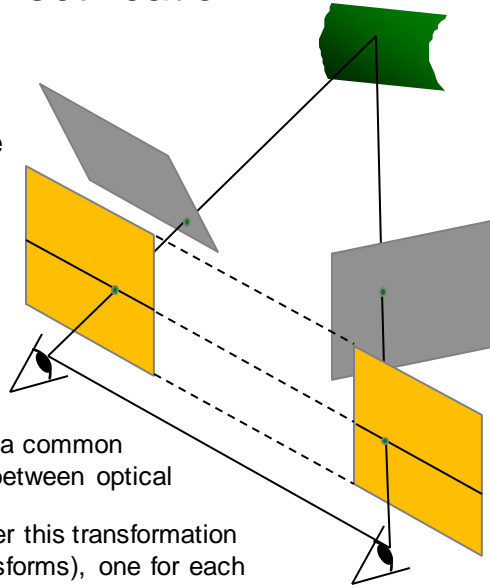


Figure from Hartley & Zisserman

Stereo image rectification

In practice, it is convenient if image scanlines (rows) are the epipolar lines.



reproject image planes onto a common plane parallel to the line between optical centers
 pixel motion is horizontal after this transformation
 two homographies (3×3 transforms), one for each input image reprojection

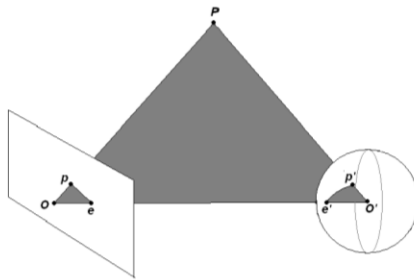
Slide credit: Li Zhang

Stereo image rectification: example



Source: Alyosha Efros

An audio camera & epipolar geometry



Spherical microphone array

Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#)
 Microphone Arrays as Generalized Cameras for Integrated Audio
 Visual Processing, IEEE Conference on Computer Vision and
 Pattern Recognition (CVPR), Minneapolis, 2007

An audio camera & epipolar geometry

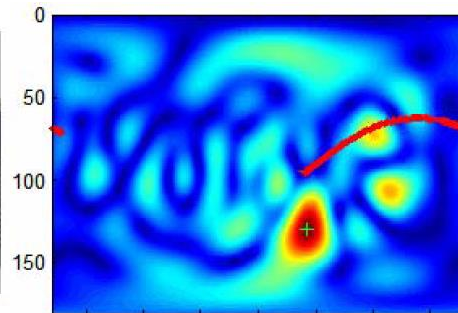


Figure 4. An example of the use of the system in speaker tracking with noise suppression. The bright red spot on the sound image (marked with a +) corresponds to the dominant source. The less dominant source however lies on the epipolar line in the sound image induced by the location of the mouth in the camera image, and this source is beamformed.

An audio camera & epipolar geometry



First without beamforming

Summary

- Depth from stereo: main idea is to triangulate from corresponding image points.
- Epipolar geometry defined by two cameras
 - We've assumed known extrinsic parameters relating their poses
- Epipolar constraint limits where points from one view will be imaged in the other
 - Makes search for correspondences quicker
- **Terms:** epipole, epipolar plane / lines, disparity, rectification, intrinsic/extrinsic parameters, essential matrix, baseline

Correspondence problem

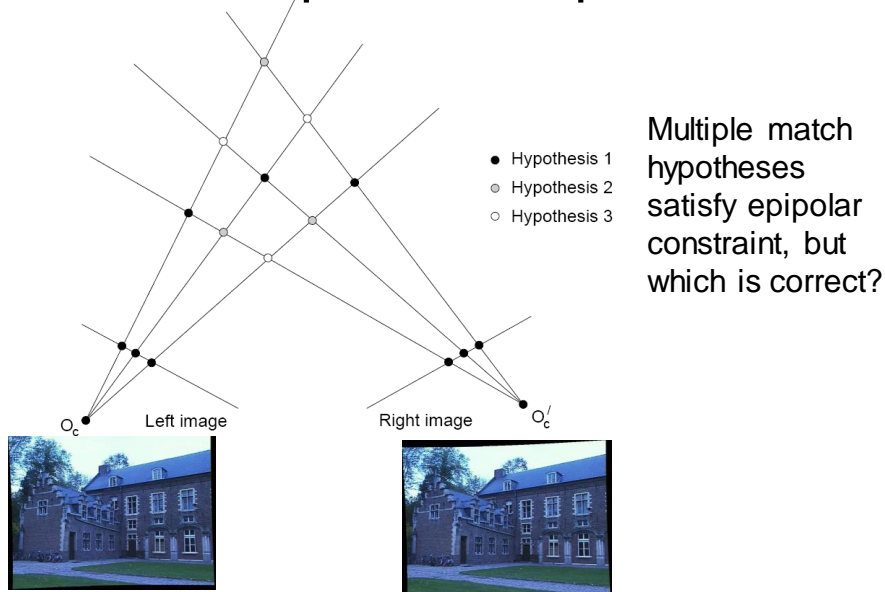
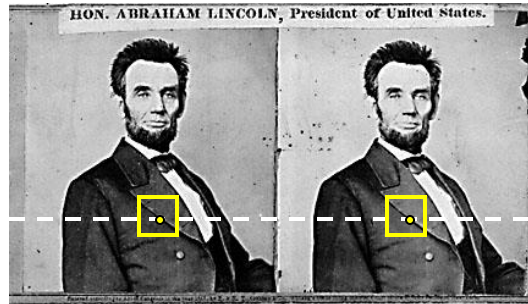


Figure from Gee & Cipolla 1999

Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
 - Similarity
 - Uniqueness
 - Ordering
 - Disparity gradient
- To find matches in the image pair, we will assume
 - Most scene points visible from both views
 - Image regions for the matches are similar in appearance

Dense correspondence search



For each epipolar line

For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

Adapted from Li Zhang

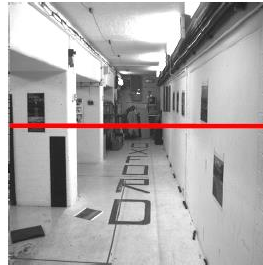
Correspondence problem



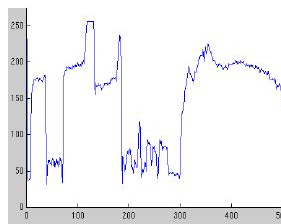
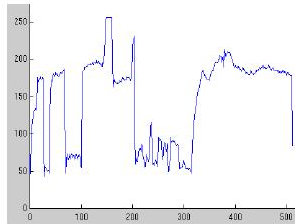
Parallel camera example: epipolar lines are corresponding image scanlines

Source: Andrew Zisserman

Correspondence problem



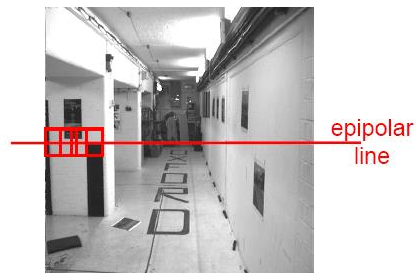
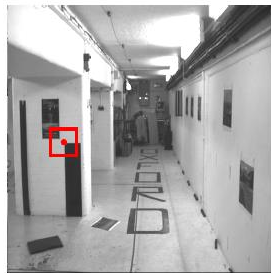
Intensity
profiles



- Clear correspondence between intensities, but also noise and ambiguity

Source: Andrew Zisserman

Correspondence problem

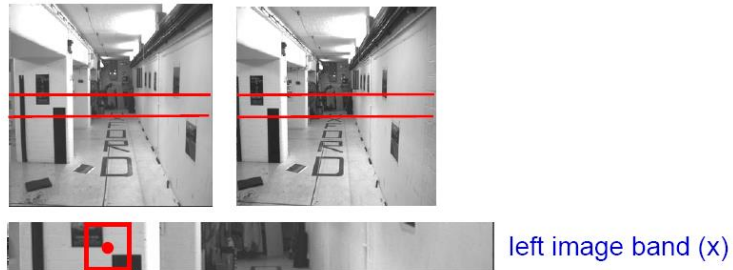


epipolar
line

Neighborhoods of corresponding points are similar in intensity patterns.

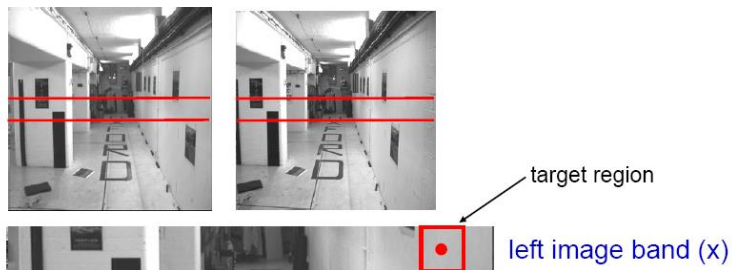
Source: Andrew Zisserman

Correlation-based window matching



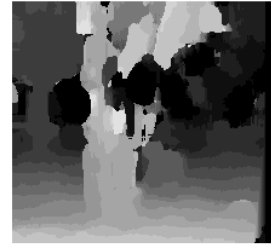
Source: Andrew Zisserman

Textureless regions



Source: Andrew Zisserman

Effect of window size



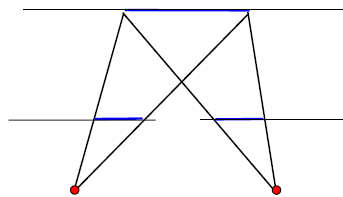
$W = 3$

$W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Figures from Li Zhang

Foreshortening effects

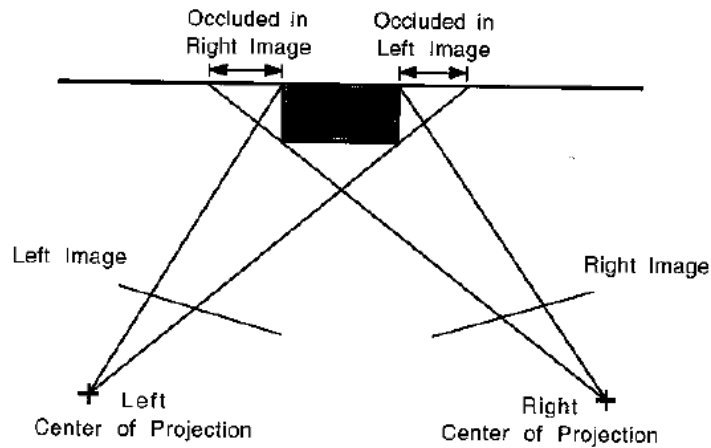


fronto-parallel surface

imaged length the same

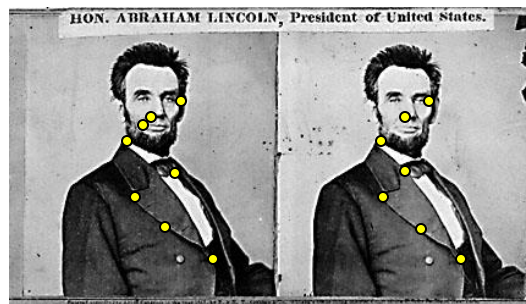
Source: Andrew Zisserman

Occlusion



Slide credit: David Kriegman

Sparse correspondence search



- Restrict search to sparse set of **detected features** (e.g., corners)
- Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
- Still narrow search further by epipolar geometry

Tradeoffs between dense and sparse search?

Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
 - Similarity
 - Uniqueness
 - Disparity gradient
 - Ordering

Uniqueness constraint

- Up to one match in right image for every point in left image

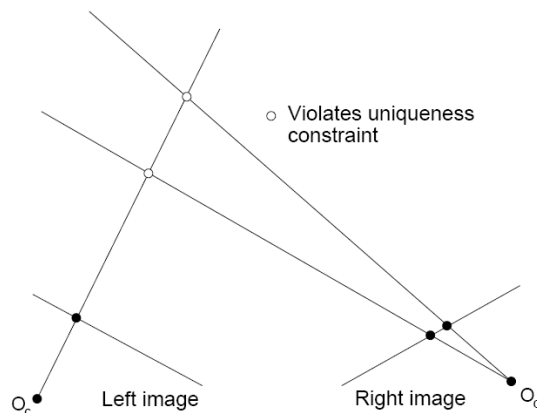
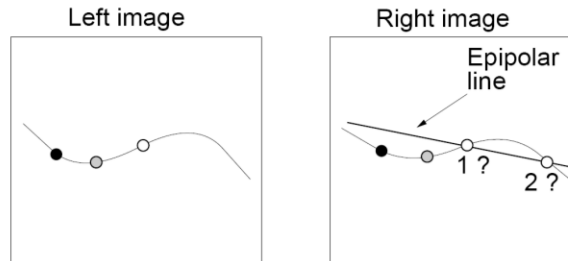


Figure from Gee &
Cipolla 1999

Disparity gradient constraint

- Assume piecewise continuous surface, so want disparity estimates to be locally smooth



Given matches ● and ●, point ○ in the left image must match point 1 in the right image. Point 2 would exceed the disparity gradient limit.

Figure from Gee & Cipolla 1999

Ordering constraint

- Points on **same surface** (opaque object) will be in same order in both views

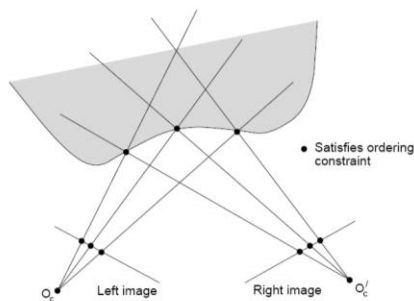
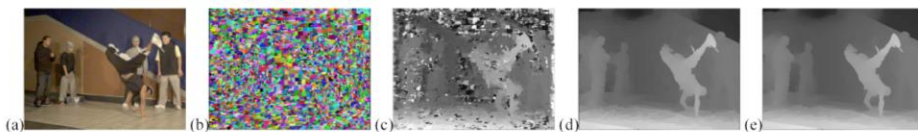
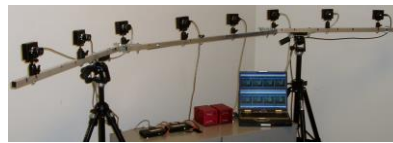


Figure from Gee & Cipolla 1999

Error sources

- Low-contrast ; textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

Virtual viewpoint video



(a) Figure 6: Sample results from stereo reconstruction stage: (a) input color image; (b) color-based segmentation; (c) initial disparity estimates \hat{d}_{ij} ; (d) refined disparity estimates; (e) smoothed disparity estimates $d_i(x)$.
 (d) A depth-matted object from earlier in the sequence is inserted into the video.

C. Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.

Virtual viewpoint video



Massive Arabesque

<http://research.microsoft.com/IVMWWW/>

Summary

- Depth from stereo: main idea is to triangulate from corresponding image points.
- Epipolar geometry defined by two cameras
 - We've assumed known extrinsic parameters relating their poses
- Epipolar constraint limits where points from one view will be imaged in the other
 - Makes search for correspondences quicker
- To estimate depth
 - Limit search by epipolar constraint
 - Compute correspondences, incorporate matching preferences