

## Survey feedback

- Topics/coverage
  - Mostly positive, enjoy content
  - More machine learning
    - More coming! We needed to build up core bg.
  - Show more cutting-edge research demos
- Exam (only a couple commented)
  - Midterm seemed long
  - A suggestion for 3 exams

- **Lecture / in-class sessions:**
  - Most find pace is good
  - Participation and discussion style
    - Like openness to questions/discussion, though sometimes derails / too specific questions (multiple comments on this)
    - Hesitation by some to ask questions for fear of classmates' response
    - Don't like participation grade requires talking (NB: it doesn't)
  - Effective tools during lecture
    - Like the review questions; do more.
    - How about a 5 minute break midway through?
    - More student interaction?
  - Classroom itself
    - Classroom gets cold sometimes
    - Classroom is in Burdine

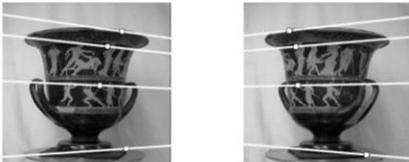
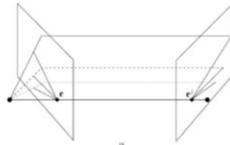
## Survey feedback

- **Website / logistics**
  - Want exam/assignment dates posted sooner
    - All available since beginning of term – see webpage
  - Make slides available sooner?
    - They are posted night before
  - Pdf vs. ppt files
    - See note on homepage
  - Suggestion to put answers to questions on slides online

## Survey feedback

- Assignments
  - Mostly positive comments, enjoyable and right level of difficulty, know where to start.
  - Couple find writeup part tedious/long
  - Hard to know when your solution is “good enough”
  - Some dislike Matlab, would prefer choice of language
  - One suggestion for more programming heavy assignments

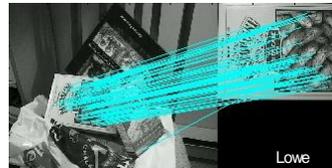
## Multiple views



Hartley and Zisserman



Multi-view geometry,  
matching, invariant  
features, stereo vision



## Why multiple views?

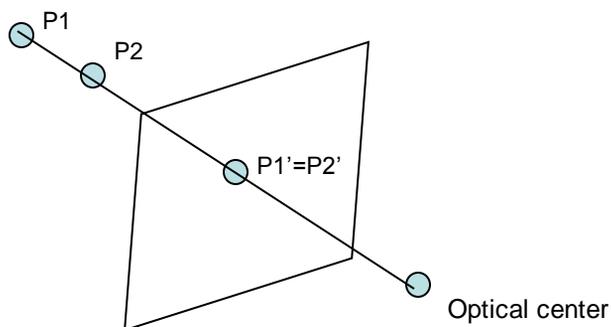
- Structure and depth are inherently ambiguous from single views.



Images from Lana Lazebni

## Why multiple views?

- Structure and depth are inherently ambiguous from single views.



## Stereo vision



Two cameras, simultaneous views

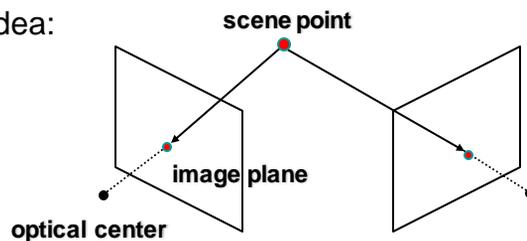


Single moving camera and static scene

## Stereo vision

- **Stereo:**
  - shape from “motion” between two views
  - infer 3d shape of scene from two (multiple) images from different viewpoints

Main idea:

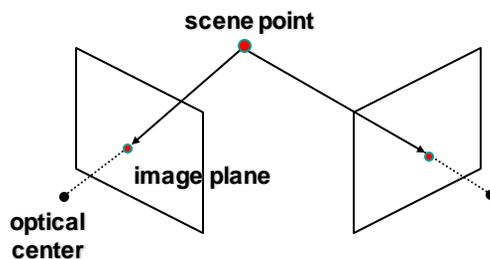


# Outline

- **Last time:**
  - Human stereopsis
  - Stereo geometry case example with parallel optical axes
- Epipolar geometry and the epipolar constraint
  - General case with calibrated cameras
- Stereo solutions
  - Correspondences
  - Additional constraints

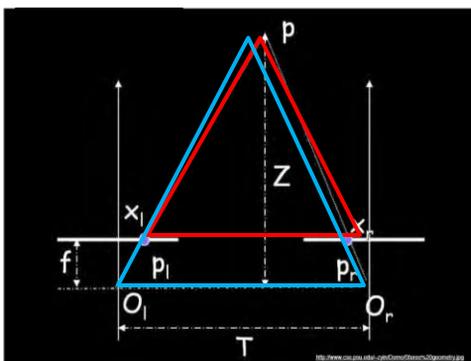
## Estimating depth with stereo

- **Stereo:** shape from “motion” between two views
- We’ll need to consider:
  - Info on camera pose (“calibration”)
  - Image point correspondences



## Recall: Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles  $(p_l, P, p_r)$  and  $(O_l, P, O_r)$ :

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_r - x_l}$$

disparity  $\rightarrow$

$$x_r - x_l$$

## Depth from disparity

image  $I(x,y)$



Disparity map  $D(x,y)$

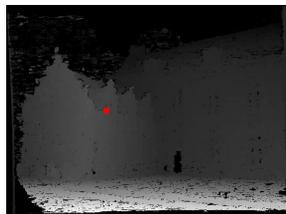


image  $I'(x',y')$

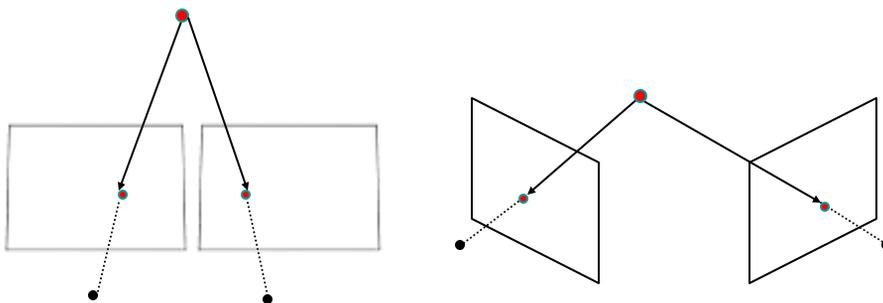


$$(x', y') = (x + D(x, y), y)$$

So if we could find the **corresponding points** in two images, we could **estimate relative depth**...

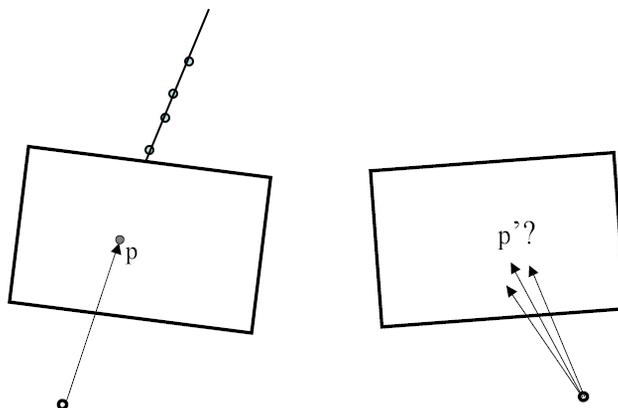
## General case, with calibrated cameras

- The two cameras need not have parallel optical axes.



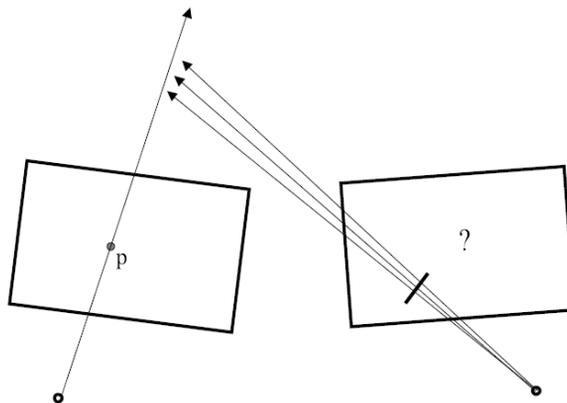
Vs.

## Stereo correspondence constraints

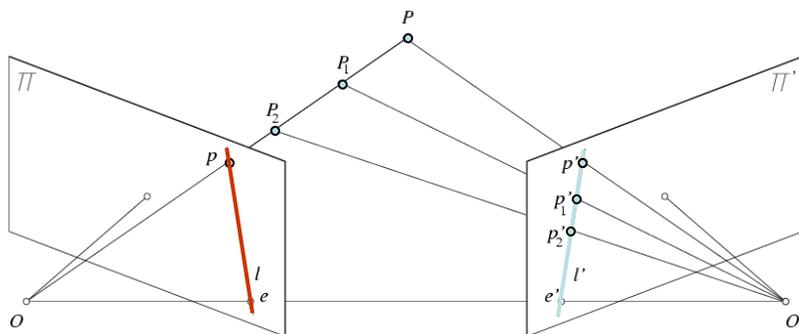


- Given  $p$  in left image, where can corresponding point  $p'$  be?

## Stereo correspondence constraints



## Epipolar constraint



Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view.

- It must be on the line carved out by a plane connecting the world point and optical centers.

## Epipolar constraint

- Potential matches for  $p$  have to lie on the corresponding epipolar line  $l'$ .
- Potential matches for  $p'$  have to lie on the corresponding epipolar line  $l$ .

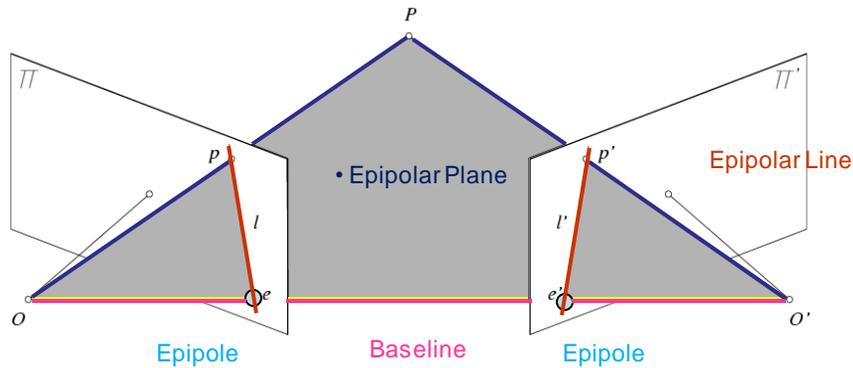
Slide credit: M. Pollefeys

## Epipolar constraint

This is useful because it reduces the correspondence problem to a 1D search along an epipolar line.

Image from Andrew Zisseman

## Epipolar geometry



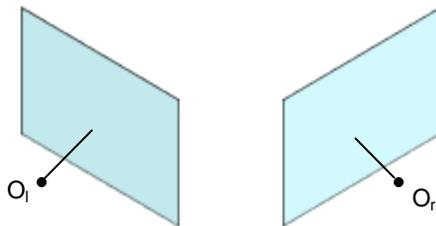
<http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html>

## Epipolar geometry: terms

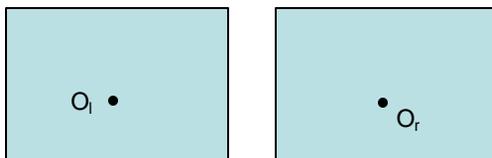
- **Baseline:** line joining the camera centers
  - **Epipole:** point of intersection of baseline with image plane
  - **Epipolar plane:** plane containing baseline and world point
  - **Epipolar line:** intersection of epipolar plane with the image plane
- 
- All epipolar lines intersect at the epipole
  - An epipolar plane intersects the left and right image planes in epipolar lines

What do the epipolar lines look like?

1.



2.



Example: converging cameras

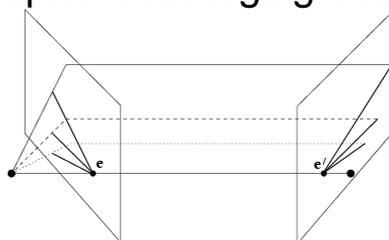


Figure from Hartley & Zisserman

## Example: parallel cameras



Where are the epipoles?

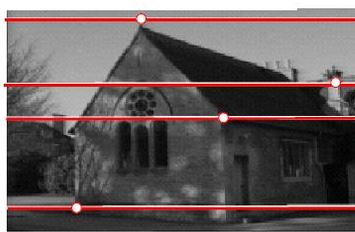
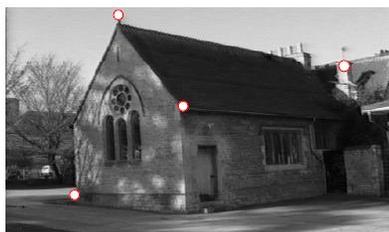
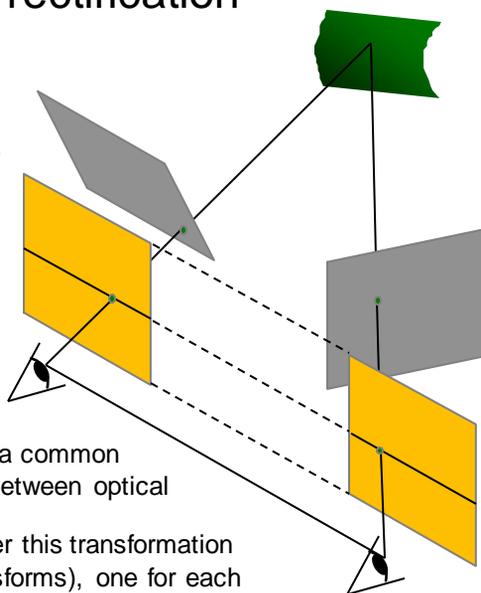


Figure from Hartley & Zisserman

## Stereo image rectification

In practice, it is convenient if image scanlines (rows) are the epipolar lines.

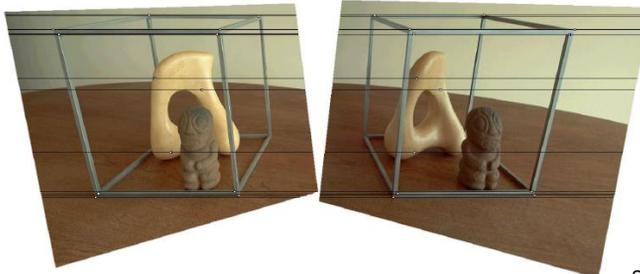


reproject image planes onto a common plane parallel to the line between optical centers

pixel motion is horizontal after this transformation  
two homographies ( $3 \times 3$  transforms), one for each input image reprojected

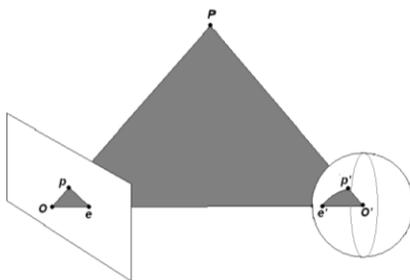
Slide credit: Li Zhang

## Stereo image rectification: example



Source: Alyosha Efros

## An audio camera & epipolar geometry



Spherical microphone array

Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#)  
 Microphone Arrays as Generalized Cameras for Integrated Audio  
 Visual Processing, IEEE Conference on Computer Vision and  
 Pattern Recognition (CVPR), Minneapolis, 2007

## An audio camera & epipolar geometry

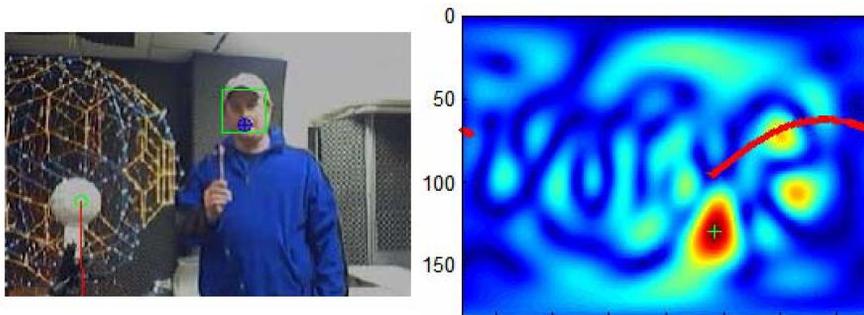
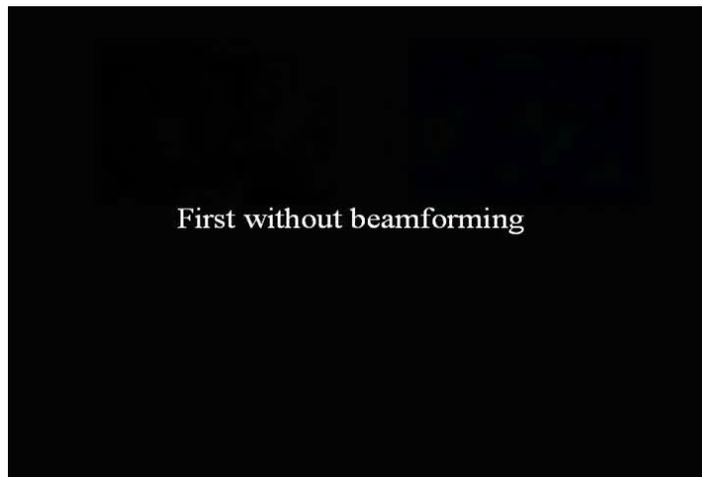


Figure 4. An example of the use of the system in speaker tracking with noise suppression. The bright red spot on the sound image (marked with a +) corresponds to the dominant source. The less dominant source however lies on the epipolar line in the sound image induced by the location of the mouth in the camera image, and this source is beamformed.

## An audio camera & epipolar geometry



Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#) Microphone Arrays as Generalized Cameras for Integrated Audio Visual Processing, IEEE Conference on Computer Vision and Pattern Recognition (CVPR), Minneapolis, 2007

## Correspondence problem

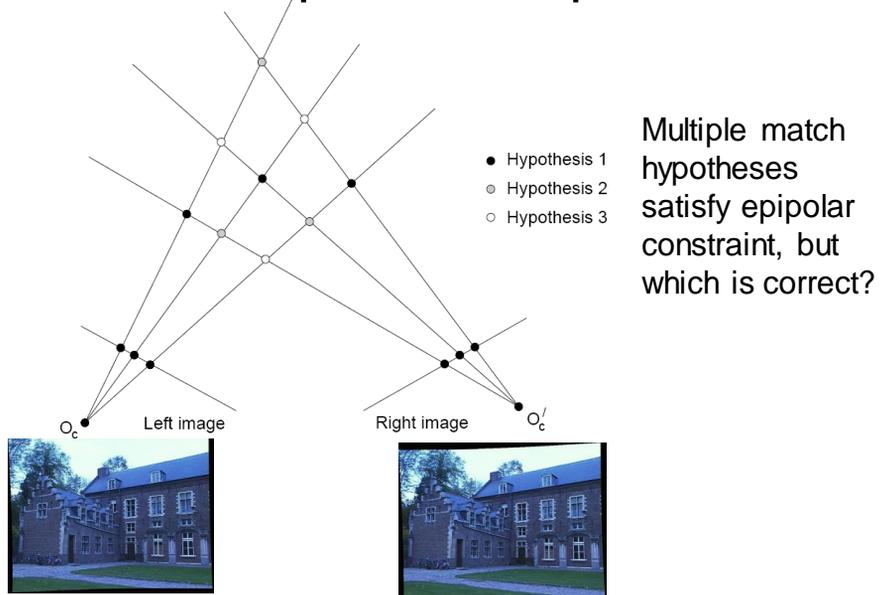
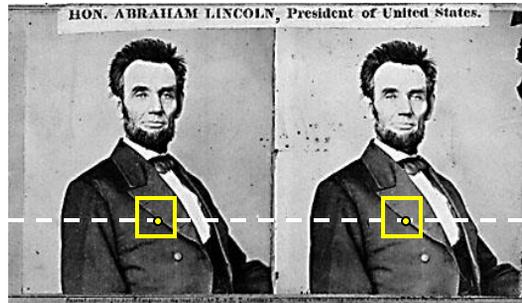


Figure from Gee &amp; Cipolla 1999

## Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Ordering
  - Disparity gradient
- To find matches in the image pair, we will assume
  - Most scene points visible from both views
  - Image regions for the matches are similar in appearance

## Dense correspondence search



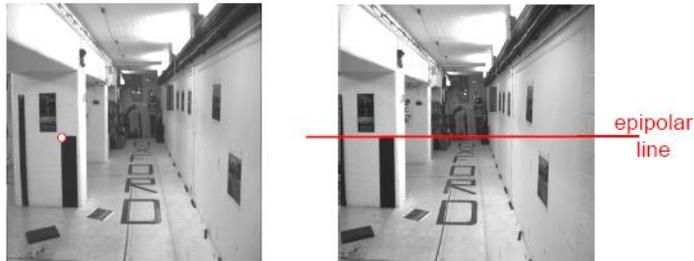
For each epipolar line

For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

Adapted from Li Zhang

## Correspondence problem



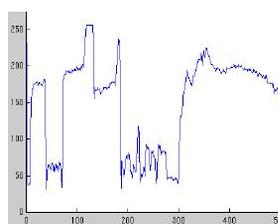
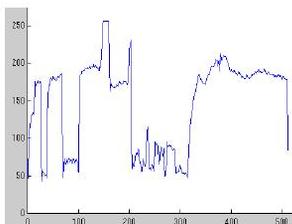
Parallel camera example: epipolar lines are corresponding image scanlines

Source: Andrew Zisserman

## Correspondence problem



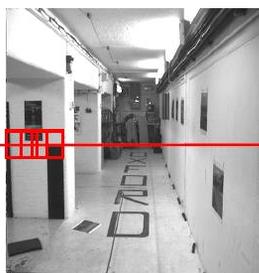
Intensity profiles



- Clear correspondence between intensities, but also noise and ambiguity

Source: Andrew Zisserman

## Correspondence problem



epipolar line

Nighborhoods of corresponding points are similar in intensity patterns.

Source: Andrew Zisserman

## Normalized cross correlation

subtract mean:  $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

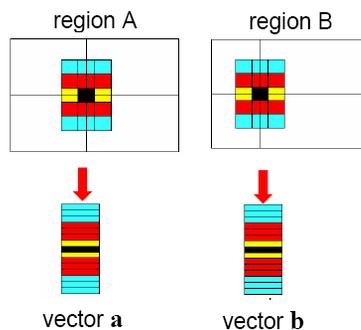
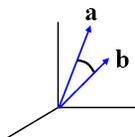
$$\text{NCC} = \frac{\sum_i \sum_j A(i, j) B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors

$A \rightarrow \mathbf{a}, B \rightarrow \mathbf{b}$

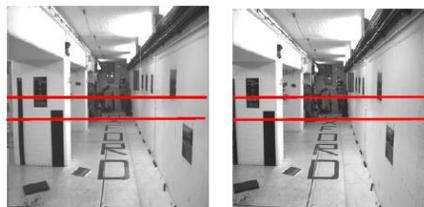
$$\text{NCC} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$$-1 \leq \text{NCC} \leq 1$$



Source: Andrew Zisserman

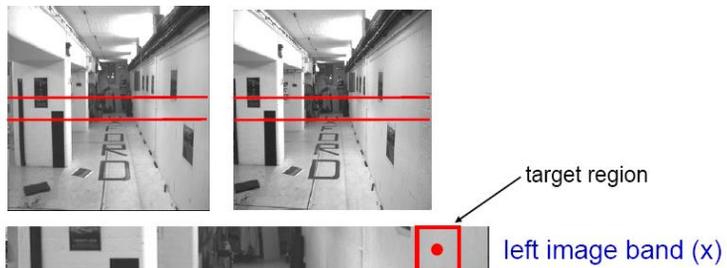
## Correlation-based window matching



left image band (x)

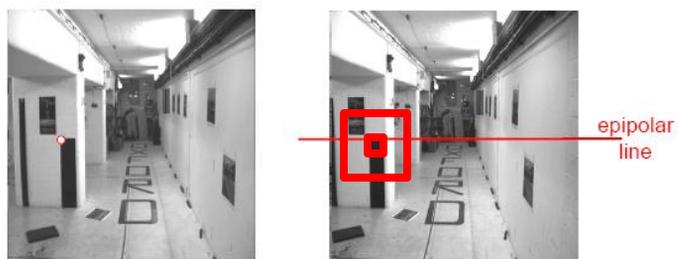
Source: Andrew Zisserman

## Textureless regions



Source: Andrew Zisserman

## Effect of window size?

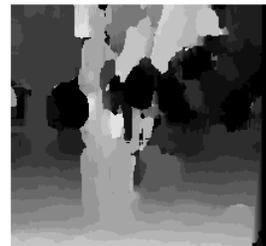


Source: Andrew Zisserman

## Effect of window size



$W = 3$

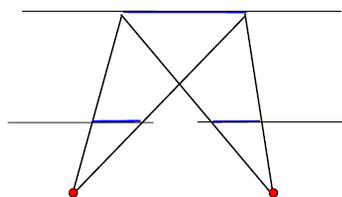


$W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Figures from Li Zhang

## Foreshortening effects

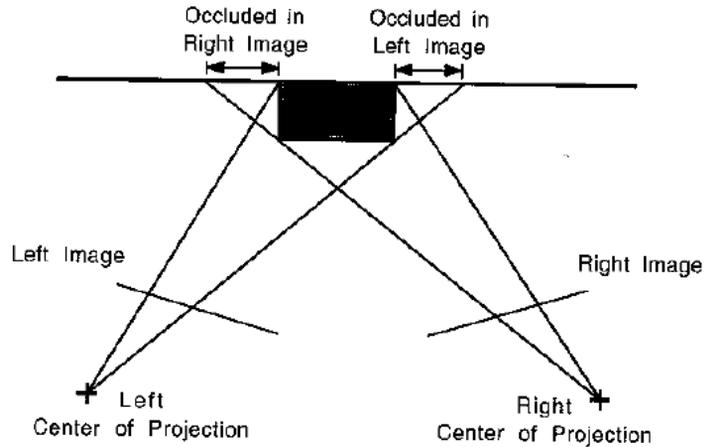


fronto-parallel surface

imaged length the same

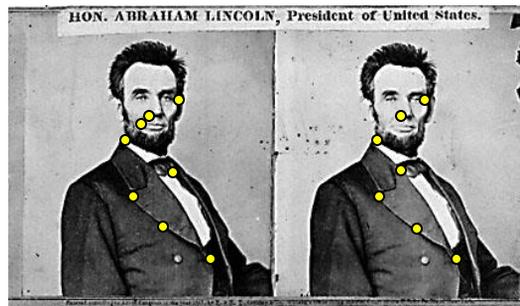
Source: Andrew Zisserman

# Occlusion



Slide credit: David Kriegman

# Sparse correspondence search



- Restrict search to sparse set of **detected features** (e.g., corners)
- Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
- Still narrow search further by epipolar geometry

*Tradeoffs between dense and sparse search?*

## Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Disparity gradient
  - Ordering

## Uniqueness constraint

- Up to one match in right image for every point in left image

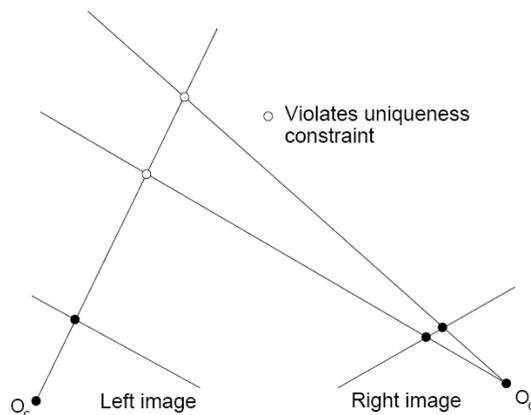
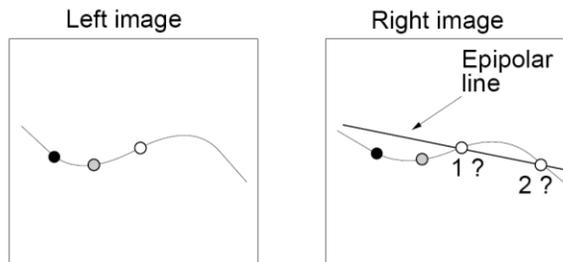


Figure from Gee & Cipolla 1999

## Disparity gradient constraint

- Assume piecewise continuous surface, so want disparity estimates to be locally smooth



Given matches ● and ○, point ○ in the left image must match point 1 in the right image. Point 2 would exceed the disparity gradient limit.

Figure from Gee & Cipolla 1999

## Ordering constraint

- Points on **same surface** (opaque object) will be in same order in both views

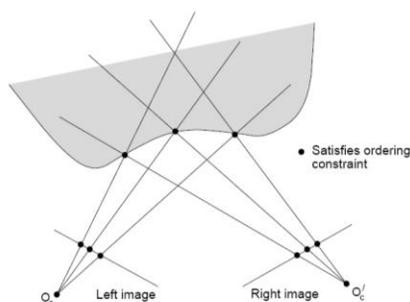


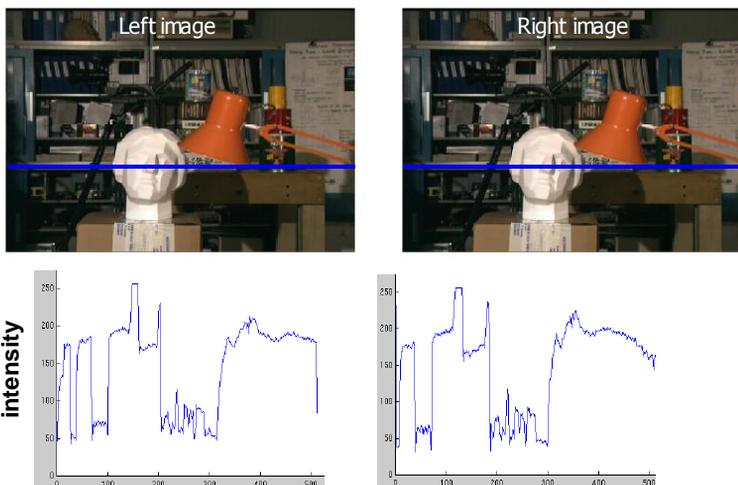
Figure from Gee & Cipolla 1999

- Beyond individual correspondences to estimate disparities:
- Optimize correspondence assignments jointly
  - Scanline at a time (DP)
  - Full 2D grid (graph cuts)

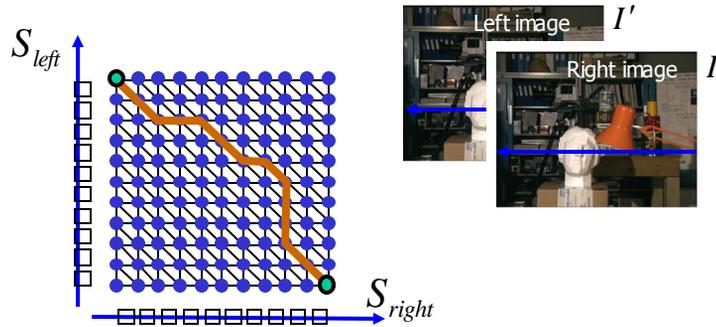
## Scanline stereo

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- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently



## “Shortest paths” for scan-line stereo

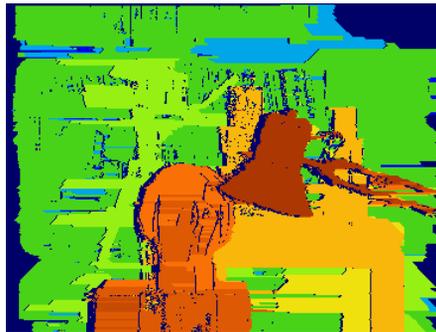


Can be implemented with dynamic programming  
Ohta & Kanade '85, Cox et al. '96

Slide credit: Y. Boykov

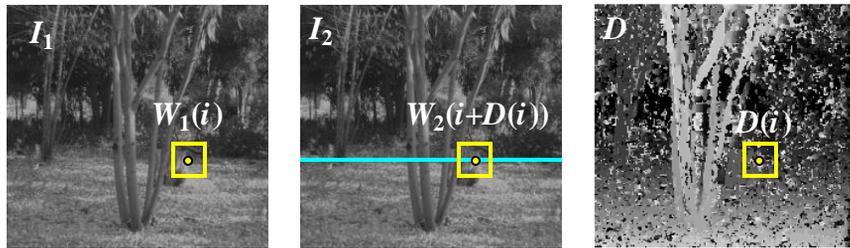
## Coherent stereo on 2D grid

- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid

## Stereo matching as energy minimization

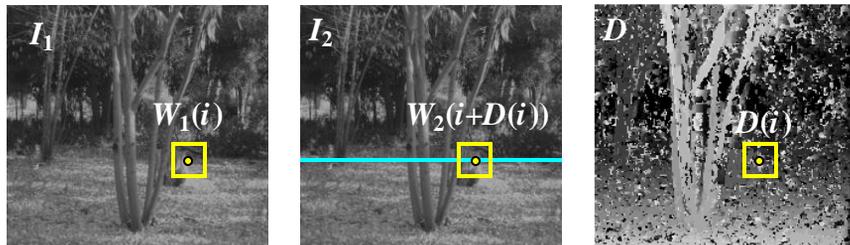


$$E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)$$

$$E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2$$

$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

## Stereo matching as energy minimization



$$E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)$$

$$E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2$$

$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

Source: Steve Seitz

## Error sources

- Low-contrast ; textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

## Depth for segmentation

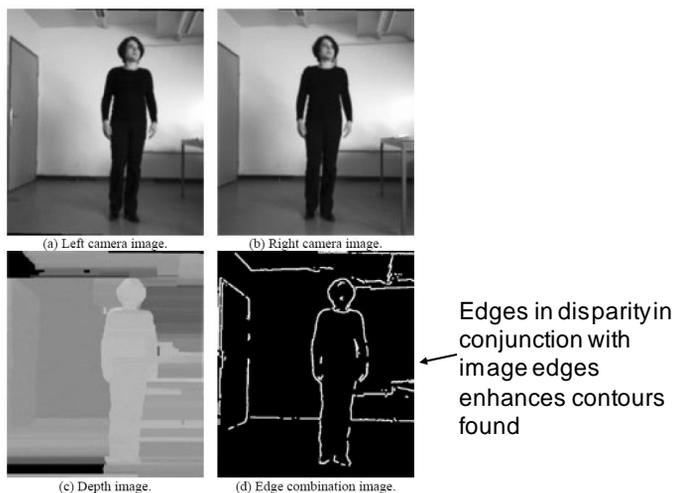
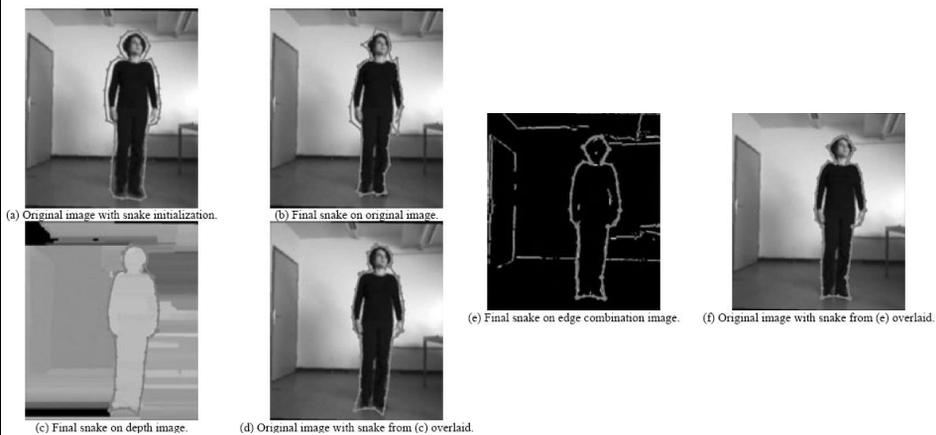


Figure 3 Stereo video frames with computed depth map and edge combination result.

Danijela Marković and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

## Depth for segmentation



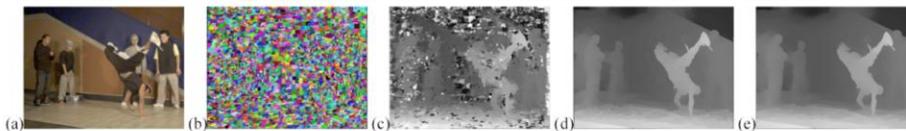
Danijela Markovic and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

## Model-based body tracking, stereo input



David Demirdjian, MIT Vision Interface Group  
<http://people.csail.mit.edu/demirdji/movie/artic-tracker/turn-around.m1v>

## Virtual viewpoint video



(a) Figure 6: Sample results from stereo reconstruction stage: (a) input color image; (b) color-based segmentation; (c) initial disparity estimates  $\hat{d}_{ij}$ ; (d) refined disparity estimates; (e) smoothed disparity estimates  $d_s(x)$ .  
 (d) A depth-matted object from earlier in the sequence is inserted into the video.

C. Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.

## Virtual viewpoint video



C. Larry Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.

<http://research.microsoft.com/IVM/VV/>

## Summary

- Depth from stereo: main idea is to triangulate from corresponding image points.
- Epipolar geometry defined by two cameras
  - We've assumed known extrinsic parameters relating their poses
- Epipolar constraint limits where points from one view will be imaged in the other
  - Makes search for correspondences quicker
- To estimate depth
  - Limit search by epipolar constraint
  - Compute correspondences, incorporate matching preferences

## Coming up

- Instance recognition
  - Indexing local features efficiently
  - Spatial verification models

