Image warping and stitching

• Midterm is Thursday
  – Covers all material up to and including last Thurs
  – Closed book, 1 sheet notes allowed
  – No coding, no Matlab

Last time

• Feature-based alignment
  – 2D transformations
  – Affine fit
  – RANSAC
Robust feature-based alignment

- Extract features

- Compute putative matches
Robust feature-based alignment

- Extract features
- Compute putative matches
- Loop:
  - Hypothesize transformation $T$ (small group of putative matches that are related by $T$)
  - Verify transformation (search for other matches consistent with $T$)

Source: L. Lazebnik
RANSAC for line fitting example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat hypothesize-and-verify loop

Last time: RANSAC for fitting a model (line)...

What about fitting a transformation (e.g., translation)?

RANSAC: General form

• RANSAC loop:
  1. Randomly select a seed group on which to base transformation estimate (e.g., a group of matches)
  2. Compute transformation from seed group
  3. Find inliers to this transformation
  4. If the number of inliers is sufficiently large, re-compute estimate of transformation on all of the inliers

• Keep the transformation with the largest number of inliers
RANSAC example: Translation

Putative matches

Select one match, count inliers

Source: Rick Szeliski
RANSAC example: Translation

Find "average" translation vector

RANSAC pros and cons

• Pros
  • Simple and general
  • Applicable to many different problems
  • Often works well in practice
• Cons
  • Parameters to tune
  • Doesn't work well for low inlier ratios (too many iterations, or can fail completely)
  • Can't always get a good initialization of the model based on the minimum number of samples

Today

• Image mosaics
  – Fitting a 2D transformation
    • Affine, Homography
  – 2D image warping
  – Computing an image mosaic
Mosaics

Obtain a wider angle view by combining multiple images.

Main questions

**Alignment:** Given two images, what is the transformation between them?

**Warping:** Given a source image and a transformation, what does the transformed output look like?

### 2D Affine Transformations

\[
\begin{bmatrix}
    x' \\
    y' \\
    w'
\end{bmatrix} =
\begin{bmatrix}
    a & b & c \\
    d & e & f \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    w
\end{bmatrix}
\]

Affine transformations are combinations of…

- Linear transformations, and
- Translations

Parallel lines remain parallel.
Projective Transformations

\[
\begin{bmatrix}
    x' \\
    y' \\
\end{bmatrix} =
\begin{bmatrix}
    a & b & c \\
    d & e & f \\
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
\end{bmatrix}
\begin{bmatrix}
    g & h & i \\
\end{bmatrix}
\begin{bmatrix}
    w \\
\end{bmatrix}
\]

Projective transformations:
- Affine transformations, and
- Projective warps

Parallel lines do not necessarily remain parallel

2D transformation models

- Similarity (translation, scale, rotation)
- Affine
- Projective (homography)

How to stitch together a panorama (a.k.a. mosaic)?

- Basic Procedure
  - Take a sequence of images from the same position
    - Rotate the camera about its optical center
  - Compute transformation between second image and first
  - Transform the second image to overlap with the first
  - Blend the two together to create a mosaic
  - (If there are more images, repeat)
- …but wait, why should this work at all?
  - What about the 3D geometry of the scene?
  - Why aren’t we using it?

Source: Steve Seitz
Pinhole camera

- Pinhole camera is a simple model to approximate imaging process, perspective projection.

If we treat pinhole as a point, only one ray from any given point can enter the camera.

Mosaics

Obtain a wider angle view by combining multiple images.

Mosaics: generating synthetic views

Can generate any synthetic camera view as long as it has the same center of projection!

Source: Alyosha Efros
The mosaic has a natural interpretation in 3D
• The images are reprojected onto a common plane
• The mosaic is formed on this plane
• Mosaic is a synthetic wide-angle camera

Source: Steve Seitz

Basic question
• How to relate two images from the same camera center?
  – how to map a pixel from PP1 to PP2

Answer
• Cast a ray through each pixel in PP1
• Draw the pixel where that ray intersects PP2

Observation:
Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image to another.

Source: Alyosha Efros

A projective transform is a mapping between any two PPs with the same center of projection
• rectangle should map to arbitrary quadrilateral
• parallel lines aren’t
• but must preserve straight lines called Homography

Source: Alyosha Efros
To compute the homography given pairs of corresponding points in the images, we need to set up an equation where the parameters of $H$ are the unknowns:

$$
\begin{pmatrix}
wx' \\
wz' \\
w
\end{pmatrix} =
\begin{pmatrix}
a & b & c & x \\
d & e & f & y \\
g & h & i & 1
\end{pmatrix}
\begin{pmatrix}
x, y
\end{pmatrix}
$$

Can set scale factor $w=1$. So, there are 8 unknowns.

Set up a system of linear equations:

$$Ah = b$$

where vector of unknowns $h = [a,b,c,d,e,f,g,h]^T$.

Need at least 8 eqs, but the more the better… Solve for $h$. If overconstrained, solve using least-squares:

$$
\min ||Ah - b||
$$

>> help ldivide

To apply a given homography $H$:

- Compute $p' = Hp$ (regular matrix multiply).
- Convert $p'$ from homogeneous to image coordinates.

$$
\begin{pmatrix}
wx' \\
wz' \\
w
\end{pmatrix} =
\begin{pmatrix}
* & * & * & x \\
* & * & * & y \\
* & * & * & 1
\end{pmatrix}
\begin{pmatrix}
x, y
\end{pmatrix}
$$
RANSAC for estimating homography

RANSAC loop:
1. Select four feature pairs (at random)
2. Compute homography $H$
3. Compute inliers where $SSD(p_i', Hp_i)<\varepsilon$
4. Keep largest set of inliers
5. Re-compute least-squares $H$ estimate on all of the inliers

Today

• Image mosaics
  – Fitting a 2D transformation
    • Affine, Homography
  – 2D image warping
    – Computing an image mosaic

Image warping

Given a coordinate transform and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?
Forward warping

Send each pixel \( f(x,y) \) to its corresponding location \( (x',y') = T(x,y) \) in the second image

Q: what if pixel lands “between” two pixels?

A: distribute color among neighboring pixels \( (x',y') \)
— Known as “splatting”

Inverse warping

Get each pixel \( g(x',y') \) from its corresponding location \( (x,y) = T^{-1}(x',y') \) in the first image

Q: what if pixel comes from “between” two pixels?
Inverse warping

Get each pixel \( g(x', y') \) from its corresponding location 
\( (x, y) = T^{-1}(x', y') \) in the first image

Q: what if pixel comes from “between” two pixels?
A: Interpolate color value from neighbors
   — nearest neighbor, bilinear...

Bilinear interpolation

Sampling at \( f(x, y) \):

\[
\begin{align*}
(i, j) & \quad f(i, j) \\
(i + 1, j) & \quad f(i + 1, j) \\
(i, j + 1) & \quad f(i, j + 1) \\
(i + 1, j + 1) & \quad f(i + 1, j + 1)
\end{align*}
\]

\[
f(x, y) = (1 - a)(1 - b) \cdot f(c, d) + a(1 - b) \cdot f(c + 1, d) + ab \cdot f(c + 1, d + 1) + (1 - a)b \cdot f(c, d + 1)
\]

Recall: generating synthetic views

Can generate any synthetic camera view as long as it has the same center of projection!

Source: Alyosha Efros
Recap: How to stitch together a panorama (a.k.a. mosaic)?

• Basic Procedure
  – Take a sequence of images from the same position
  – Rotate the camera about its optical center
  – Compute transformation (homography) between second image and first using corresponding points.
  – Transform the second image to overlap with the first.
  – Blend the two together to create a mosaic.
  – (If there are more images, repeat)

Source: Steve Seitz

Image warping with homographies

Image rectification
Analysing patterns and shapes

What is the shape of the black and white floor pattern?

The floor (enlarged)

Automatically rectified floor

Slide from Antonio Criminisi

From Martin Kemp, *The Science of Art*

(manual reconstruction)

Analysing patterns and shapes

What is the (complicated) shape of the floor pattern?

Automatically rectified floor

*St. Lucy Altarpiece, D. Veneziano*

Slide from Criminisi
Analysing patterns and shapes

Automatic rectification

From Martin Kemp, *The Science of Art* (manual reconstruction)

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Changing camera center

Does it still work?

Does it still work?

Can generate synthetic camera view as long as it has the same center of projection.

Source: Alyosha Efros
…Or: Planar scene (or far away)

PP3 is a projection plane of both centers of projection, so we are OK!
This is how big aerial photographs are made

Boundary extension

Creating and Exploring a Large Photorealistic Virtual Space

Summary: alignment & warping

- **Write 2d transformations** as matrix-vector multiplication (including translation when we use homogeneous coordinates)
- Perform **image warping** (forward, inverse)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.