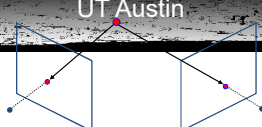


## Stereo

Thurs Mar 23  
Kristen Grauman  
UT Austin



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### Previously

- Write **2d transformations** as matrix-vector multiplication
- Perform **image warping** (forward, inverse)
- **Fitting transformations**: solve for unknown parameters given corresponding points from two views (affine, projective (homography)).
- **Mosaics**: uses homography and image warping to merge views taken from same center of projection.

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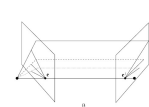
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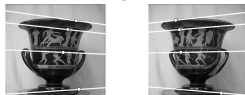
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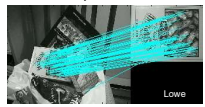
### Multiple views



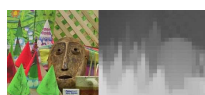
Multi-view geometry,  
matching, invariant  
features, stereo vision




Hartley and Zisserman



Lowe





Kristen Grauman

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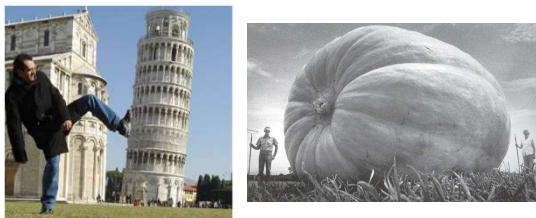
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### Why multiple views?

- Structure and depth are inherently ambiguous from single views.



Images from Lana Lazebnik

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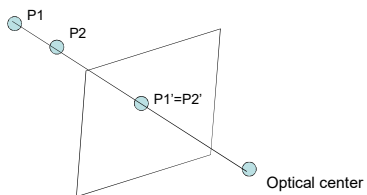
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### Why multiple views?

- Structure and depth are inherently ambiguous from single views.



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- What cues help us to perceive 3d shape and depth?

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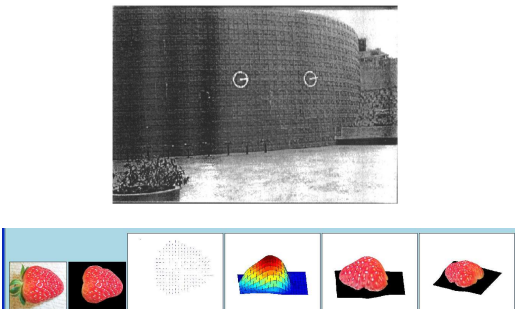
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### Texture



[From A.M. Loh. The recovery of 3-D structure using visual texture patterns. PhD thesis]

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### Perspective effects



Image credit: S. Seltz

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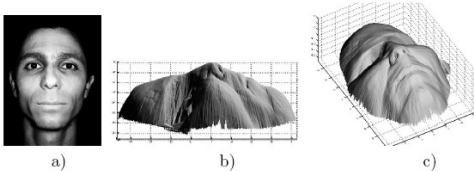
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### Shading



[Figure from Prados & Faugeras 2006]

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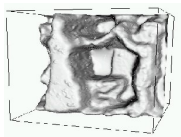
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### Focus/defocus



Images from same point of view, different camera parameters





3d shape / depth estimates

[figs from H. Jin and P. Favaro, 2002]

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### Motion





Figures from L. Zhang <http://www.brainconnection.com/teasers/?main=illusion/motion-shape>

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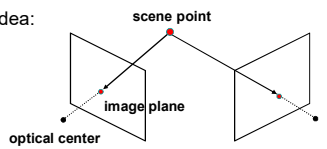
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### Estimating scene shape

- "Shape from X": Shading, Texture, Focus, Motion...
- **Stereo:**
  - shape from "motion" between two views
  - infer 3d shape of scene from two (multiple) images from different viewpoints

Main idea:



Kristen Grauman

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## Outline

- Human stereopsis
- Epipolar geometry and the epipolar constraint
  - Case example with parallel optical axes
  - General case with calibrated cameras
- Stereo solutions
  - Correspondences
  - Additional constraints

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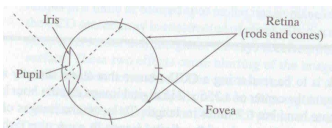
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## Human eye



Pupil/Iris – control amount of light passing through lens  
Retina - contains sensor cells, where image is formed  
Fovea – highest concentration of cones

Fig from Shapiro and Stockman

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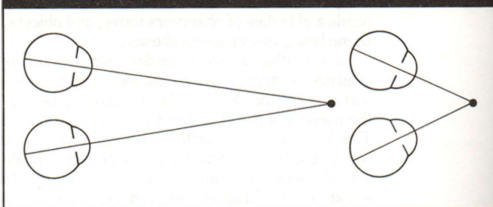
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## Human stereopsis: disparity

FIGURE 7.1



From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Human eyes **fixate** on point in space – rotate so that corresponding images form in centers of fovea.

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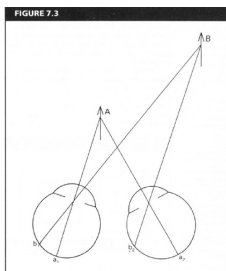
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### Human stereopsis: disparity



**Disparity** occurs when eyes fixate on one object; others appear at different visual angles

From Bruce and Green, Visual Perception, Physiology, Psychology and Ecology

Adapted from David Forsyth, UC Berkeley

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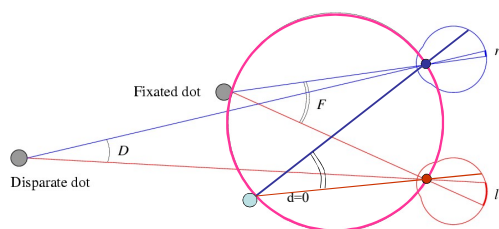
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### Human stereopsis: disparity



Disparity:  $d = r-l = D-F$

Forsyth & Ponce

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### Random dot stereograms

- Julesz 1960: Do we identify local brightness patterns before fusion (monocular process) or after (binocular)?
- To test: pair of synthetic images obtained by randomly spraying black dots on white objects

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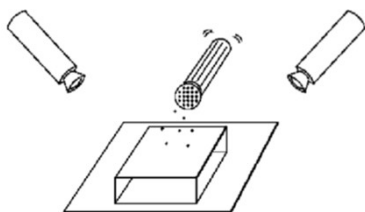
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### Random dot stereograms



Forsyth & Ponce

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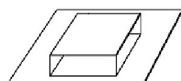
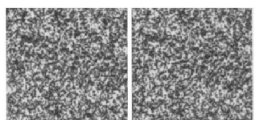
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### Random dot stereograms



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### Random dot stereograms

- When viewed monocularly, they appear random; when viewed stereoscopically, see 3d structure.
- Conclusion: human binocular fusion not directly associated with the physical retinas; must involve the central nervous system
- Imaginary "*cyclopean retina*" that combines the left and right image stimuli as a single unit

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## Stereo photography and stereo viewers

Take two pictures of the same subject from two slightly different viewpoints and display so that each eye sees only one of the images.



Invented by Sir Charles Wheatstone, 1838



Image from fisher-price.com

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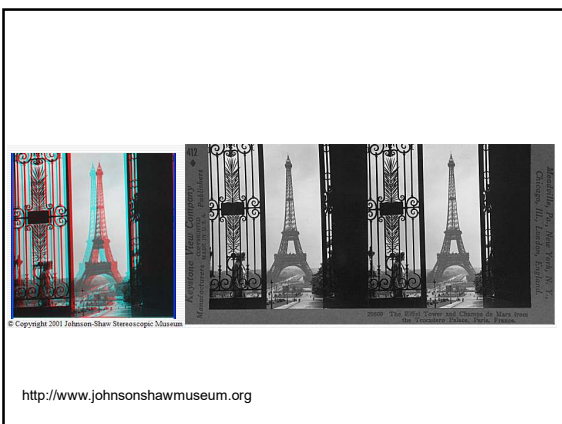
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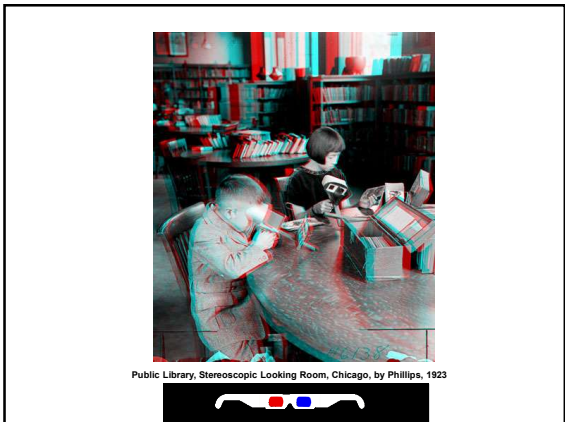
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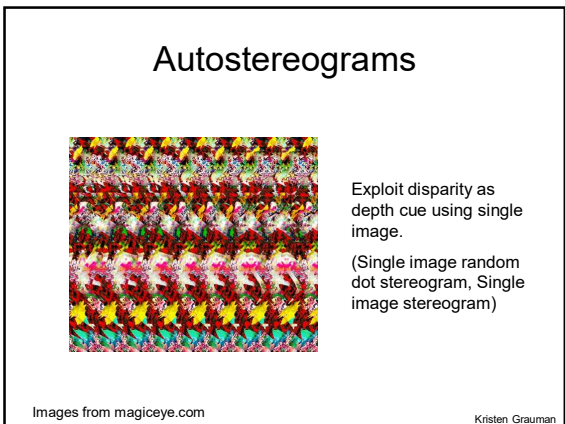
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## Autostereograms



Images from magic-eye.com

Kristen Grauman

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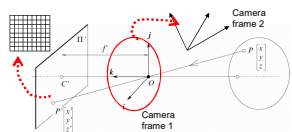
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## Camera parameters



**Extrinsic parameters:**  
Camera frame 1  $\leftrightarrow$  Camera frame 2

**Intrinsic parameters:**  
Image coordinates relative to camera  $\leftrightarrow$  Pixel coordinates

- *Extrinsic* params: rotation matrix and translation vector
- *Intrinsic* params: focal length, pixel sizes (mm), image center point, radial distortion parameters

*We'll assume for now that these parameters are given and fixed.*

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## Outline

- Human stereopsis
- Stereograms
- Epipolar geometry and the epipolar constraint
  - Case example with parallel optical axes
  - General case with calibrated cameras

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### Stereo vision



Two cameras, simultaneous views



Single moving camera and static scene

Kristen Grauman

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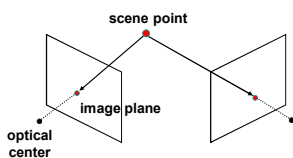
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### Estimating depth with stereo

- **Stereo:** shape from "motion" between two views
- We'll need to consider:
  - Info on camera pose ("calibration")
  - Image point correspondences



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### Geometry for a simple stereo system

- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):

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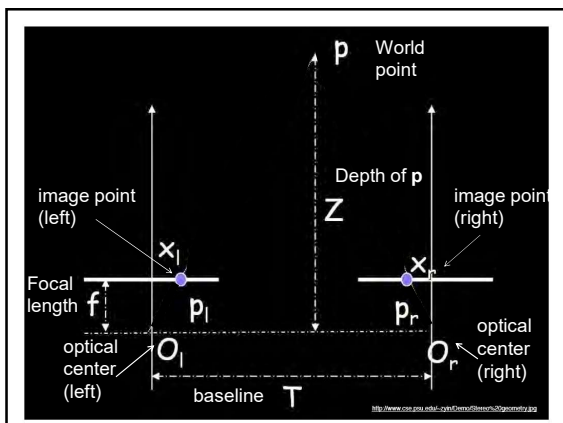
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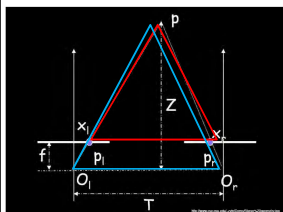
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### Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). **What is expression for Z?**



Similar triangles  $(p_l, P, p_r)$  and  $(O_l, P, O_r)$ :

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_r - x_l}$$

disparity  $\rightarrow$

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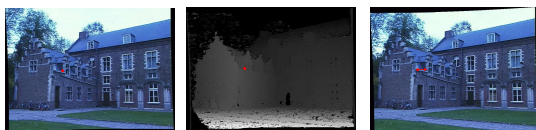
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### Depth from disparity

image  $I(x,y)$

Disparity map  $D(x,y)$

image  $I'(x',y')$



$$(x',y') = (x + D(x,y), y)$$

So if we could find the **corresponding points** in two images, we could **estimate relative depth**...

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## Outline

- Human stereopsis
- Stereograms
- Epipolar geometry and the epipolar constraint
  - Case example with parallel optical axes
  - General case with calibrated cameras

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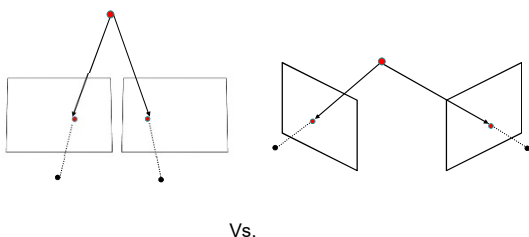
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## General case, with calibrated cameras

- The two cameras need not have parallel optical axes.




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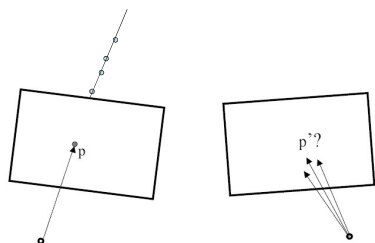
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## Stereo correspondence constraints



- Given  $p$  in left image, where can corresponding point  $p'$  be?

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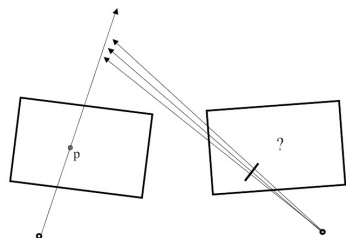
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### Stereo correspondence constraints




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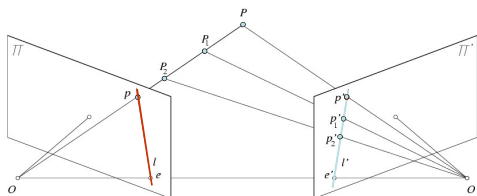
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### Epipolar constraint



Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view.

- It must be on the line carved out by a plane connecting the world point and optical centers.

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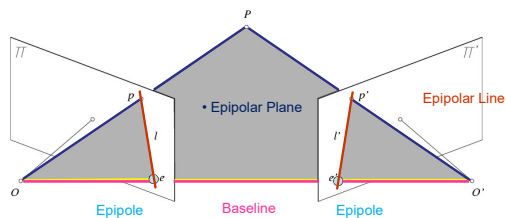
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### Epipolar geometry



<http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html>

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### Epipolar geometry: terms

- **Baseline:** line joining the camera centers
  - **Epipole:** point of intersection of baseline with image plane
  - **Epipolar plane:** plane containing baseline and world point
  - **Epipolar line:** intersection of epipolar plane with the image plane
- 
- All epipolar lines intersect at the epipole
  - An epipolar plane intersects the left and right image planes in epipolar lines

*Why is the epipolar constraint useful?*

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### Epipolar constraint



This is useful because it reduces the correspondence problem to a 1D search along an epipolar line.

Image from Andrew Zisserman

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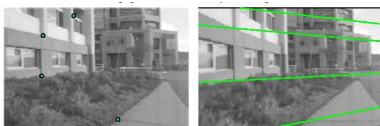
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### Example



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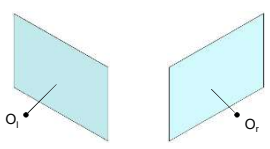
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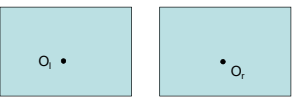
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What do the epipolar lines look like?

1. 

2. 

Kristen Grauman

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Example: converging cameras

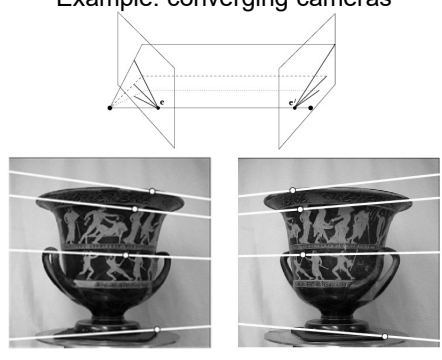


Figure from Hartley & Zisserman

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Example: parallel cameras

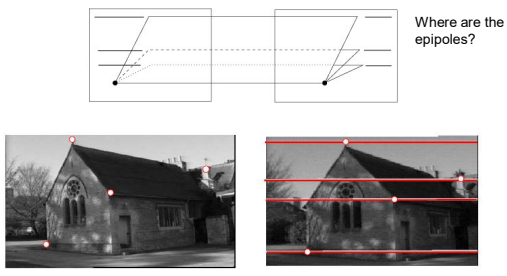


Figure from Hartley & Zisserman

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### Stereo image rectification

In practice, it is convenient if image scanlines (rows) are the epipolar lines.

reproject image planes onto a common plane parallel to the line between optical centers

pixel motion is horizontal after this transformation

two homographies (3x3 transforms), one for each input image reproject

Slide credit: Li Zhang

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### Stereo image rectification: example

Source: Alyosha Efros

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### An audio camera & epipolar geometry

Spherical microphone array

Adam O' Donovan, [Ramani Duraiswami](#) and [Jan Neumann](#)  
 Microphone Arrays as Generalized Cameras for Integrated Audio Visual Processing, IEEE Conference on Computer Vision and Pattern Recognition (CVPR), Minneapolis, 2007

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### An audio camera & epipolar geometry

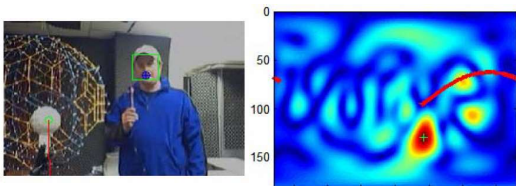


Figure 4. An example of the use of the system in speaker tracking with noise suppression. The bright red spot on the sound image (marked with a +) corresponds to the dominant source. The less dominant source however lies on the epipolar line in the sound image induced by the location of the mouth in the camera image, and this source is beamformed.

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### An audio camera & epipolar geometry




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### Summary so far

- Depth from stereo: main idea is to triangulate from corresponding image points.
- Epipolar geometry defined by two cameras
  - We've assumed known extrinsic parameters relating their poses
- Epipolar constraint limits where points from one view will be imaged in the other
  - Makes search for correspondences quicker
- **Terms:** epipole, epipolar plane / lines, disparity, rectification, baseline

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### Correspondence problem

● Hypothesis 1  
 ○ Hypothesis 2  
 ◐ Hypothesis 3

Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Figure from Geis & Cipolla 1999

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### Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Ordering
  - Disparity gradient
- To find matches in the image pair, we will assume
  - Most scene points visible from both views
  - Image regions for the matches are similar in appearance

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### Dense correspondence search

Adapted from Li Zhang

For each epipolar line

For each pixel / window in the left image

- compare with every pixel / window on same epipolar line in right image
- pick position with minimum match cost (e.g., SSD, correlation)

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### Correspondence problem



Parallel camera example: epipolar lines are corresponding image scanlines

Source: Andrew Zisserman

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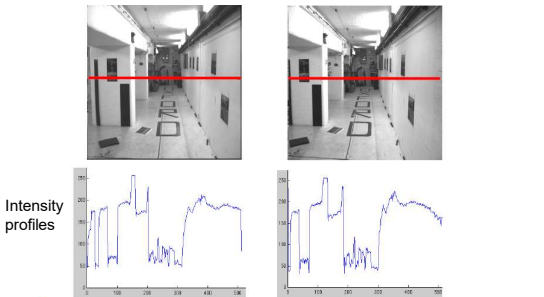
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### Correspondence problem



Intensity profiles

- Clear correspondence between intensities, but also noise and ambiguity

Source: Andrew Zisserman

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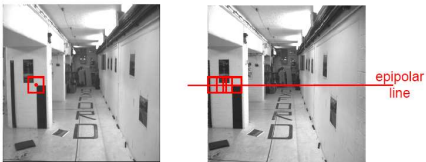
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### Correspondence problem



Neighborhoods of corresponding points are similar in intensity patterns.

Source: Andrew Zisserman

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### Normalized cross correlation

subtract mean:  $A \leftarrow A - \langle A \rangle, B \leftarrow B - \langle B \rangle$

$$NCC = \frac{\sum_i \sum_j A(i, j) B(i, j)}{\sqrt{\sum_i \sum_j A(i, j)^2} \sqrt{\sum_i \sum_j B(i, j)^2}}$$

Write regions as vectors  
 $A \rightarrow \mathbf{a}, B \rightarrow \mathbf{b}$

$$NCC = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$$

$-1 \leq NCC \leq 1$

Source: Andrew Zisserman

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### Correlation-based window matching

Source: Andrew Zisserman

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### Textureless regions

Source: Andrew Zisserman

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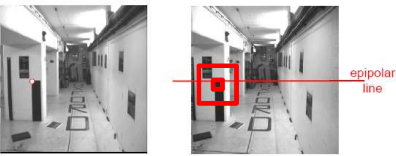
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### Effect of window size?



Source: Andrew Zisserman

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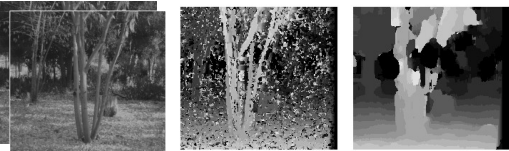
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### Effect of window size



W = 3      W = 20

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Figures from Li Zhang

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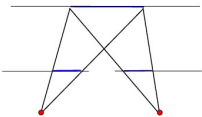
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### Foreshortening effects



fronto-parallel surface  
imaged length the same

Source: Andrew Zisserman

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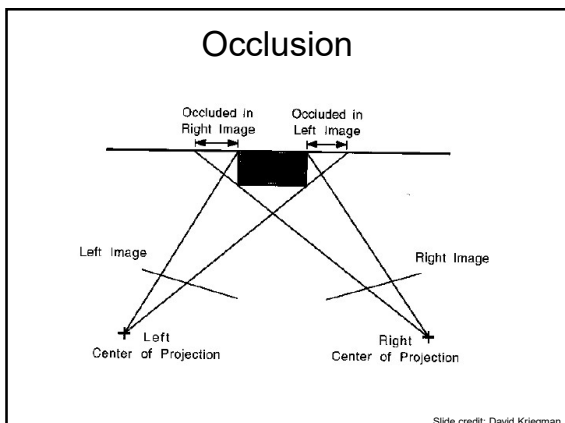
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### Sparse correspondence search

HON. ABRAHAM LINCOLN, President of United States.

- Restrict search to sparse set of **detected features** (e.g., corners)
- Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
- Still narrow search further by epipolar geometry

*Tradeoffs between dense and sparse search?*

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### Correspondence problem

- Beyond the hard constraint of epipolar geometry, there are "soft" constraints to help identify corresponding points
  - Similarity
  - Uniqueness
  - Disparity gradient
  - Ordering

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### Uniqueness constraint

- Up to one match in right image for every point in left image

Figure from Gee & Cipolla 1999

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### Disparity gradient constraint

- Assume piecewise continuous surface, so want disparity estimates to be locally smooth

Given matches ● and ○, point ○ in the left image must match point 1 in the right image. Point 2 would exceed the disparity gradient limit.

Figure from Gee & Cipolla 1999

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### Ordering constraint

- Points on **same surface** (opaque object) will be in same order in both views

Figure from Gee & Cipolla 1999

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### Ordering constraint

- Won't always hold, e.g. consider transparent object, or an occluding surface

Figures from Forsyth & Ponce

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- Beyond individual correspondences to estimate disparities:
- Optimize correspondence assignments jointly
  - Scanline at a time (DP)
  - Full 2D grid (graph cuts)

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### Scanline stereo

- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently

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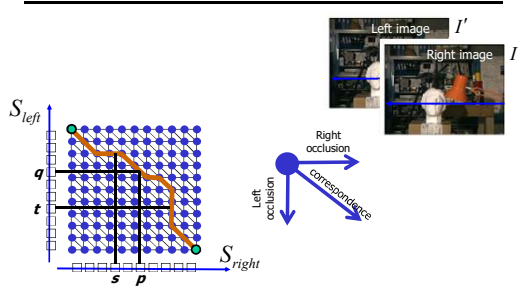
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### “Shortest paths” for scan-line stereo



Can be implemented with dynamic programming  
Ohta & Kanade '85, Cox et al. '96

Slide credit: Y. Boykov

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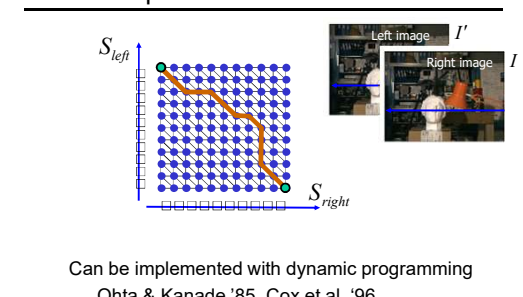
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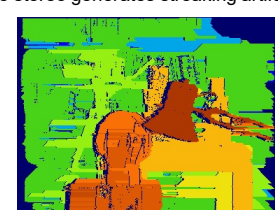
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### Coherent stereo on 2D grid

- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid

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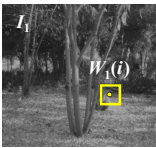
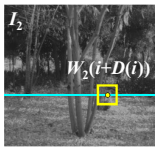
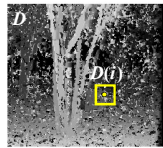
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**Stereo matching as energy minimization**

$$E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D)$$

$$E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2$$

$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

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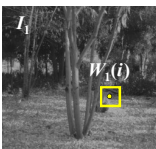
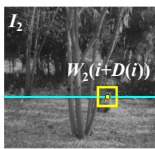
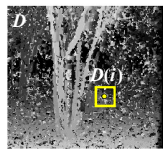
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**Stereo matching as energy minimization**

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$$E_{\text{smooth}} = \sum_{\text{neighbors } i, j} \rho(D(i) - D(j))$$

- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

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**Error sources**

- Low-contrast ; textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

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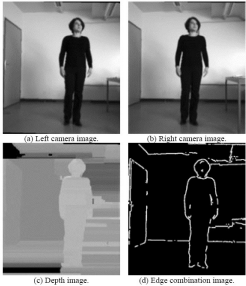
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### Depth for segmentation



(a) Left camera image (b) Right camera image  
(c) Depth image (d) Edge combination image

Edges in disparity in conjunction with image edges enhances contours found

Figure 3 Stereo video frames with computed depth map and edge combination result.

Danjela Markovic and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

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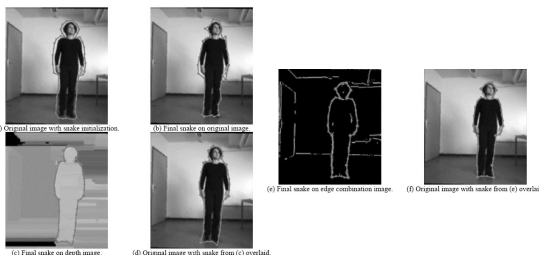
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### Depth for segmentation



(a) Original image with white mask (b) Mask on original image  
(c) Mask on depth image (d) Mask on edge combination image  
(e) Original image with mask from (c) overlaid

Danjela Markovic and Margrit Gelautz, Interactive Media Systems Group, Vienna University of Technology

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### Model-based body tracking, stereo input



David Demirdjian, MIT Vision Interface Group  
<http://people.csail.mit.edu/demirdj/movies/artic-tracker/turn-around.m1v>

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### Virtual viewpoint video














Figure 6: Sample results from stereo reconstruction stage: (a) input color image; (b) color-based segmentation; (c) initial disparity estimates  $d_0$ ; (d) refined disparity estimates; (e) smoothed disparity estimates  $d_1(x)$ .  
 A depth-mapped object from earlier in the sequence is inserted into the video.

C. Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.

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### Virtual viewpoint video



C. Larry Zitnick et al, High-quality video view interpolation using a layered representation, SIGGRAPH 2004.  
<http://research.microsoft.com/IVM/VVV/>

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### Summary

- Depth from stereo: main idea is to triangulate from corresponding image points.
- Epipolar geometry defined by two cameras
  - We've assumed known extrinsic parameters relating their poses
- Epipolar constraint limits where points from one view will be imaged in the other
  - Makes search for correspondences quicker
- To estimate depth
  - Limit search by epipolar constraint
  - Compute correspondences, incorporate matching preferences

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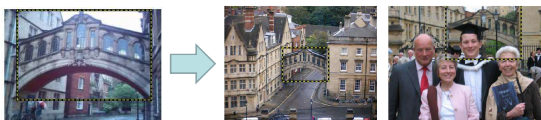
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## Coming up

- Instance recognition
  - Indexing local features efficiently
  - Spatial verification models



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