SVM wrap-up and Neural Networks

Tues April 25
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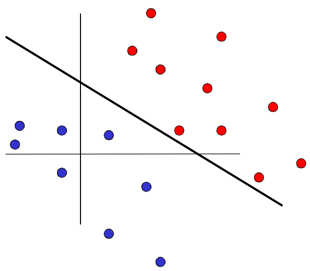
Last time

- Supervised classification continued
  - Nearest neighbors (wrap up)
  - Support vector machines
    - HoG pedestrians example
    - Understanding classifier mistakes with iHoG
    - Kernels
    - Multi-class from binary classifiers

Today

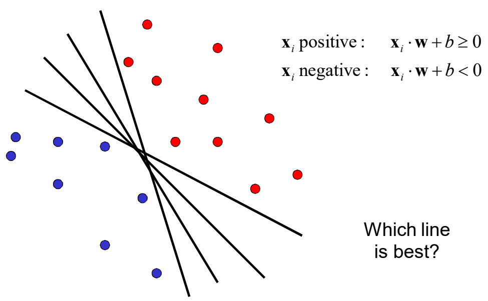
- Support vector machines (wrap-up)
  - Pyramid match kernels
- Evaluation
  - Scoring an object detector
  - Scoring a multi-class recognition system
- Intro to (deep) neural networks

Recall: Linear classifiers

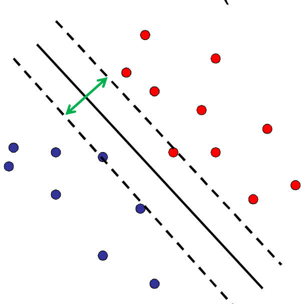


Recall: Linear classifiers

- Find linear function to separate positive and negative examples



Recall: Support Vector Machines (SVMs)



- Discriminative classifier based on *optimal separating line (for 2d case)*
- Maximize the *margin* between the positive and negative training examples

Recall: Form of SVM solution

- Solution: $\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i$

  $b = y_i - \mathbf{w} \cdot \mathbf{x}_i$ (for any support vector)

  $\mathbf{w} \cdot \mathbf{x} + b = \sum_i \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x} + b$
- Classification function:

$$f(x) = \text{sign}\,(\mathbf{w} \cdot \mathbf{x} + \text{b})$$

$$= \text{sign}\left(\sum_i \alpha_i y_i \boxed{\mathbf{x}_i \cdot \mathbf{x}} + b\right)$$

If f(x) < 0, classify as negative, if f(x) > 0, classify as positive

C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1

Nonlinear SVMs

- *The kernel trick*: instead of explicitly computing the lifting transformation $\varphi(\mathbf{x})$, define a kernel function K such that

$$K(\mathbf{x}_i, \mathbf{x}_j) = \varphi(\mathbf{x}_i) \cdot \varphi(\mathbf{x}_j)$$

- This gives a nonlinear decision boundary in the original feature space:

$$\sum_i \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}) + b$$

SVMs: Pros and cons

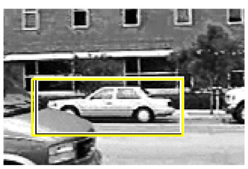
- Pros
  - Kernel-based framework is very powerful, flexible
  - Often a sparse set of support vectors – compact at test time
  - Work very well in practice, even with small training sample sizes
- Cons
  - No “direct” multi-class SVM, must combine two-class SVMs
  - Can be tricky to select best kernel function for a problem
  - Computation, memory
    - During training time, must compute matrix of kernel values for every pair of examples
    - Learning can take a very long time for large-scale problems

Adapted from Lana Lazebnik

Review questions

- What are tradeoffs between the one vs. one and one vs. all paradigms for multi-class classification?
- What roles do kernels play within support vector machines?
- What can we expect the training images associated with support vector to look like?
- What is hard negative mining?

Scoring a sliding window detector



If prediction and ground truth are *bounding boxes*, when do we have a correct detection?

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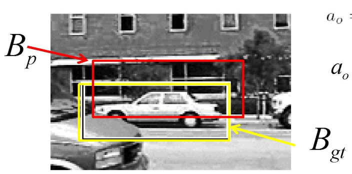
Scoring a sliding window detector



$$a_o = \frac{area(B_p \cap B_{gt})}{area(B_p \cup B_{gt})}$$

$$a_o > 0.5 \Rightarrow correct$$

We'll say the detection is correct (a "true positive") if the intersection of the bounding boxes, divided by their union, is > 50%.

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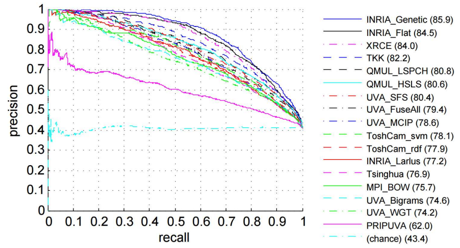
Scoring an object detector



- If the detector can produce a *confidence score* on the detections, then we can plot its precision vs. recall as a threshold on the confidence is varied.
- **Average Precision (AP)**: mean precision across recall levels.

Recalll: Examples of kernel functions

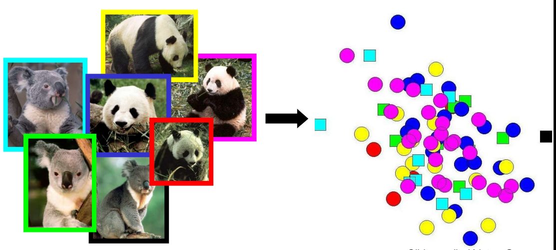
- Linear: $K(x_i, x_j) = x_i^T x_j$
- Gaussian RBF: $K(x_i,x_j) = \exp(-\frac{\|x_i - x_j\|^2}{2\sigma^2})$
- Histogram intersection:

$$K(x_i, x_j) = \sum_k \min(x_i(k), x_j(k))$$

- Kernels go beyond vector space data
- Kernels also exist for "structured" input spaces like sets, graphs, trees…

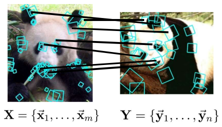
Discriminative classification with sets of features?

- Each instance is unordered set of vectors
- Varying number of vectors per instance



Slide credit: Kristen Grauman

Partially matching sets of features



Optimal match: O(m³)
Greedy match: O(m² log m)
Pyramid match: O(m)

(**m**=num pts)

$$\min_{\pi: \mathbf{X} \to \mathbf{Y}} \sum_{\mathbf{x}_i \in \mathbf{X}} ||\mathbf{x}_i - \pi(\mathbf{x}_i)||$$

nate matching kernel that makes it practical to compare large sets of features based on their partial correspondences.

[Previous work: Indyk & Thaper, Bartal, Charikar, Agarwal & Varadarajan, …]

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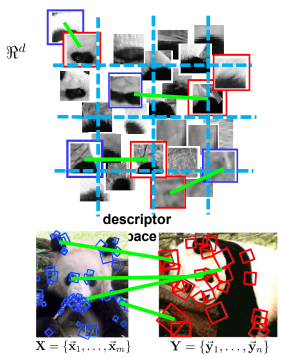
Pyramid match: main idea



Feature space partitions serve to "match" the local descriptors within successively wider regions.

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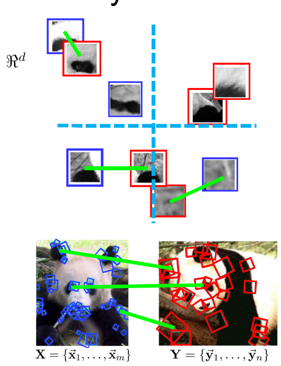
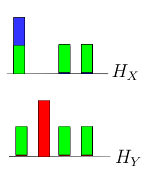
Pyramid match: main idea





$$\mathcal{I}(H_X, H_Y) = \sum_j \min(H_X(j), H_Y(j))$$
$$= 3$$

Histogram intersection counts number of possible matches at a given partitioning.

Slide credit: Kristen Grauman

Pyramid match

$$K_{\Delta}(X,Y) = \sum_{i=0}^{L} \underbrace{2^{-i}}_{\substack{\text{measures}\\ \text{difficulty of a}\\ \text{match at level } i}} \underbrace{\left\{ \mathcal{I}\left(H_X^{(i)}, H_Y^{(i)}\right) - \mathcal{I}\left(H_X^{(i-1)}, H_Y^{(i-1)}\right) \right\}}_{\substack{\text{number of newly matched}\\ \text{pairs at level } i}}$$

- For similarity, weights inversely proportional to bin size (or may be learned)
- Normalize these kernel values to avoid favoring large sets

[Grauman & Darrell, ICCV 2005]

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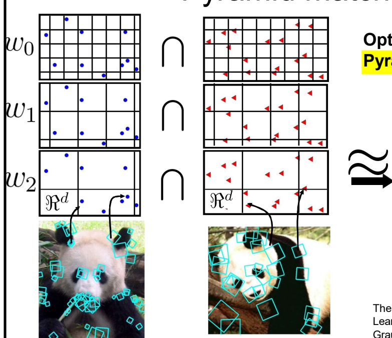
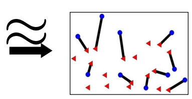
Pyramid match



Optimal match: O(m³)
Pyramid match: O(mL)

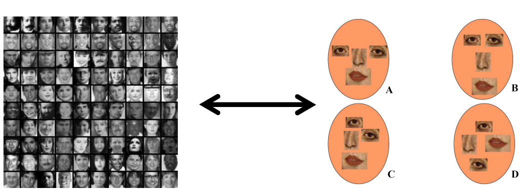


optimal partial matching

$\mathbf{X} = \{\vec{\mathbf{x}}_1, \ldots, \vec{\mathbf{x}}_m\}$ $\mathbf{Y} = \{\vec{\mathbf{y}}_1, \ldots, \vec{\mathbf{y}}_n\}$

The Pyramid Match Kernel: Efficient Learning with Sets of Features. K. Grauman and T. Darrell. Journal of Machine Learning Research (JMLR), 8 (Apr): 725--760, 2007.

BoW Issue:
No spatial layout preserved!



Too much? Too little?

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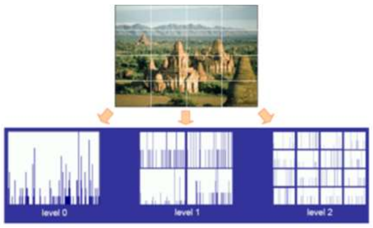
Spatial pyramid match

- Make a pyramid of bag-of-words histograms.
- Provides some loose (global) spatial layout information



[Lazebnik, Schmid & Ponce, CVPR 2006]

Spatial pyramid match

- Make a pyramid of bag-of-words histograms.
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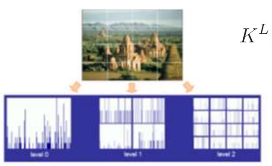


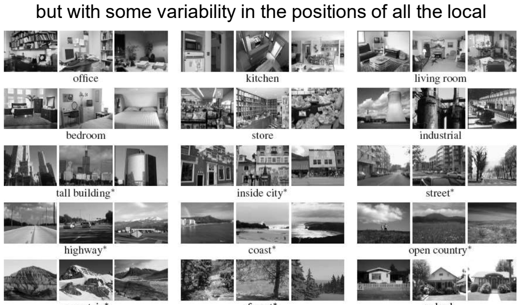
$$K^L(X,Y) = \sum_{m=1}^{M} \kappa^L(X_m, Y_m)$$

Sum over PMKs computed in *image coordinate* space, one per word.

[Lazebnik, Schmid & Ponce, CVPR 2006]

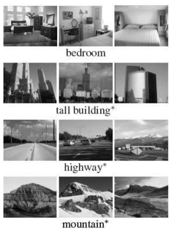
Spatial pyramid match

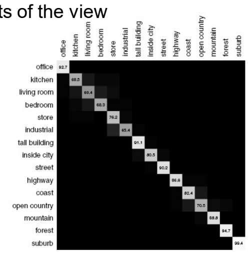
- Can capture **scene** categories well---texture-like patterns but with some variability in the positions of all the local



Spatial pyramid match

- Can capture **scene** categories well---texture-like patterns but with some variability in the positions of all the local pieces.
- Sensitive to global shifts of the view





Confusion table

Summary: Past week

- Object recognition as classification task
  - Boosting (face detection ex)
  - Support vector machines and HOG (person detection ex)
    - Pyramid match kernels
    - Hoggles visualization for understanding classifier mistakes
  - Nearest neighbors and global descriptors (scene rec ex)
- Sliding window search paradigm
  - Pros and cons
  - Speed up with attentional cascade
- Evaluation
  - Detectors: Intersection over union, precision recall
  - Classifiers: Confusion matrix

Today

- Support vector machines (wrap-up)
  - Pyramid match kernels
- Evaluation
  - Scoring an object detector
  - Scoring a multi-class recognition system
- **Intro to (deep) neural networks**

Traditional Image Categorization: Training phase

Training

Training Images

Image Features → Classifier Training → Trained Classifier

Training Labels → Classifier Training

Slide credit: Jia-Bin Huang

Traditional Image Categorization: Testing phase

Training

Training Images

Image Features → Classifier Training → Trained Classifier

Training Labels → Classifier Training

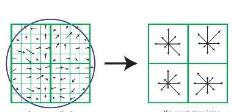
Testing

Test Image

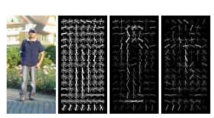
Image Features → Trained Classifier → Prediction **Outdoor**

Slide credit: Jia-Bin Huang

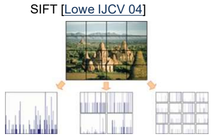
Features have been key



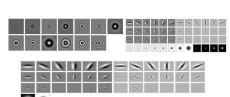
SIFT [Lowe IJCV 04]



HOG [Dalal and Triggs CVPR 05]



SPM [Lazebnik et al. CVPR 06]



Textons

and many others:

SURF, MSER, LBP, Color-SIFT, Color histogram, GLOH, …..

Learning a Hierarchy of Feature Extractors

- Each layer of hierarchy extracts features from output of previous layer
- All the way from pixels → classifier
- Layers have the (nearly) same structure

Image/video → Layer 1 → Layer 2 → Layer 3 → Labels

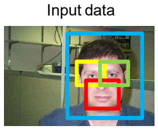
- Train all layers jointly

Slide: Rob Fergus

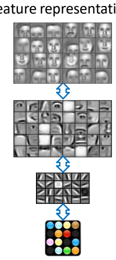
Learning Feature Hierarchy

Goal: Learn **useful higher-level features** from images

Input data



Feature representation



3rd layer
"Objects"

2nd layer
"Object parts"

1st layer
"Edges"

Pixels

Lee et al., ICML 2009;
CACM 2011

Slide: Rob Fergus

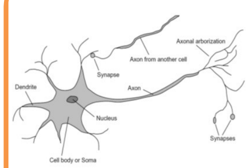
Learning Feature Hierarchy

- Better performance
- Other domains (unclear how to hand engineer):
  - Kinect
  - Video
  - Multi spectral

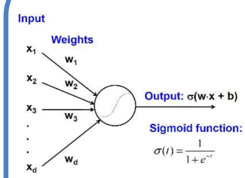
- Feature computation time
  - Dozens of features now regularly used [e.g., MKL]
  - Getting prohibitive for large datasets (10's sec /image)

Slide: R. Fergus

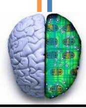
Biological neuron and Perceptrons



A biological neuron



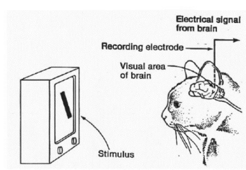
An artificial neuron (Perceptron)
- a linear classifier



Slide credit: Jia-Bin Huang

Simple, Complex and Hypercomplex cells

David H. Hubel and Torsten Wiesel

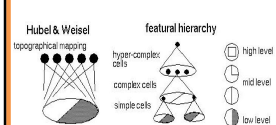


Suggested a **hierarchy** of **feature detectors** in the visual cortex, with higher level features responding to patterns of activation in lower level cells, and propagating activation upwards to still higher level cells.

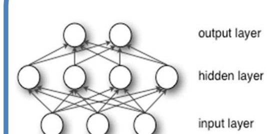
David Hubel's Eye, Brain, and Vision

Slide credit: Jia-Bin Huang

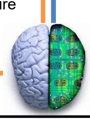
Hubel/Wiesel Architecture and Multi-layer Neural Network



Hubel and Weisel's architecture



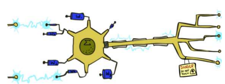
Multi-layer Neural Network
- A *non-linear* classifier



Slide credit: Jia-Bin Huang

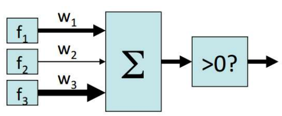
Neuron: Linear Perceptron

- Inputs are feature values
- Each feature has a weight
- Sum is the activation



$$\text{activation}_w(x) = \sum_i w_i \cdot f_i(x) = w \cdot f(x)$$

- If the activation is:
  - Positive, output +1
  - Negative, output -1



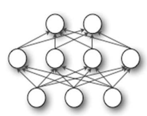
Slide credit: Pieter Abeel and Dan Klein

Multi-layer Neural Network

- A non-linear classifier
- **Training:** find network weights **w** to minimize the error between true training labels y_i and estimated labels $f_{\boldsymbol{w}}(\boldsymbol{x_i})$

$$E(\mathbf{w}) = \sum_{i=1}^{N} \left(y_i - f_{\mathbf{w}}(\mathbf{x}_i)\right)^2$$

- Minimization can be done by gradient descent provided f is differentiable
- This training method is called **back-propagation**



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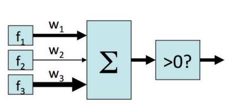
Two-layer perceptron network



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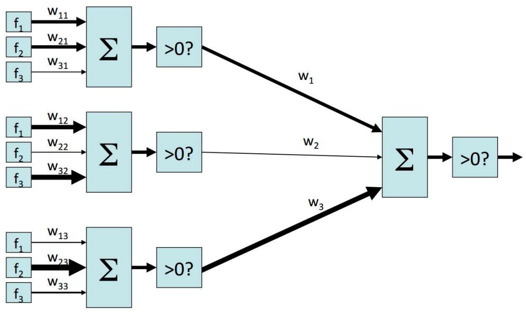
Two-layer perceptron network



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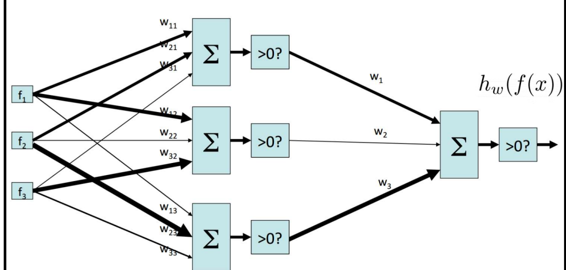
Two-layer perceptron network



Slide credit: Pieter Abeel and Dan Klein

Learning w

- Training examples

$$(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$$

- Objective: a misclassification loss

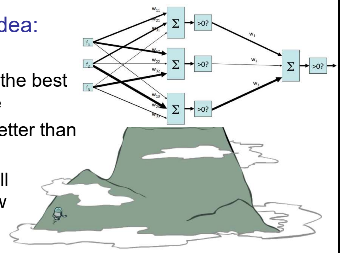
$$\min_w \sum_{i=1}^{m} \left(y^{(i)} - h_w(f(x^{(i)})) \right)^2$$

- Procedure:
  - Gradient descent / hill climbing

Slide credit: Pieter Abeel and Dan Klein

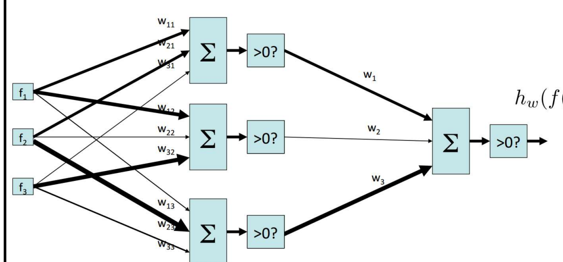
Hill climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit
  - Neighbors = small perturbations of w
- What's bad?
  - Complete?
  - Optimal?



Slide credit: Pieter Abeel and Dan Klein

Two-layer perceptron network



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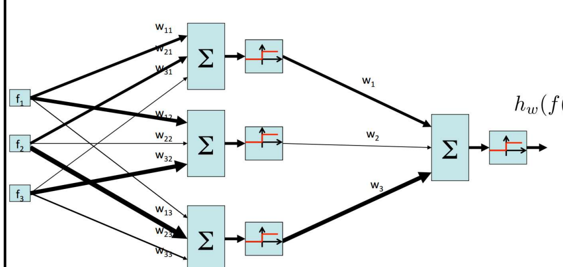
Two-layer perceptron network



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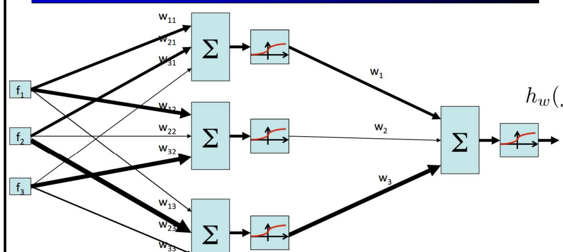
Two-layer neural network



$z \to \frac{1}{1+e^{-z}}$

Slide credit: Pieter Abeel and Dan Klein

Neural network properties

- Theorem (Universal function approximators): A two-layer network with a sufficient number of neurons can approximate any continuous function to any desired accuracy
- Practical considerations:
  - Can be seen as learning the features
  - Large number of neurons
    - Danger for overfitting
  - Hill-climbing procedure can get stuck in bad local optima

Approximation by Superpositions of Sigmoidal Function,1989

Slide credit: Pieter Abeel and Dan Klein

Recap

- Pyramid match kernels:
  - Example of structured input data for kernel-based classifiers (SVM)
- Neural networks / multi-layer perceptrons
  - View of neural networks as learning hierarchy of features

Coming up

- Convolutional neural networks for image classification