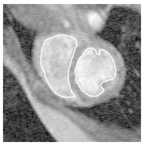
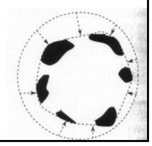


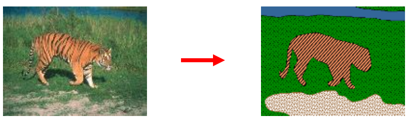

Fitting: Deformable contours

Thurs Oct 1

Kristen Grauman
UT Austin



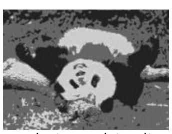
Recap so far: Grouping and Fitting



Goal: move from array of pixel values (or filter outputs) to a collection of regions, objects, and shapes.

Grouping: Pixels vs. regions


image


clusters on intensity

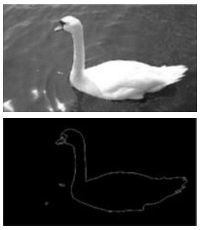

image


clusters on color

By grouping pixels based on Gestalt-inspired attributes, we can map the pixels into a set of regions.

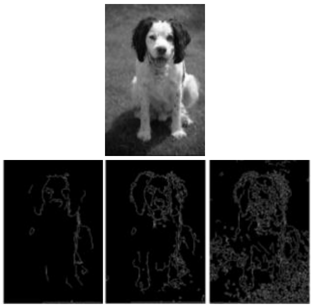
Each region is consistent according to the features and similarity metric we used to do the **clustering**.

Fitting: Edges vs. boundaries



Edges useful signal to indicate occluding boundaries, shape.

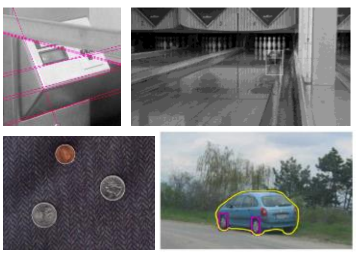
Here the raw edge output is not so bad…



…but quite often boundaries of interest are fragmented, and we have extra "clutter" edge points.

Images from D. Jacobs

Fitting: Edges vs. boundaries

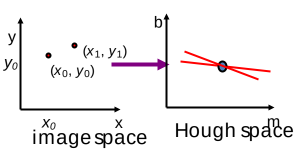


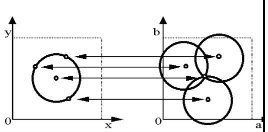
Given a model of interest, we can overcome some of the missing and noisy edges using **fitting** techniques.

With voting methods like the **Hough transform**, detected points vote on possible model parameters.

Voting with Hough transform

- Hough transform for fitting lines, circles, arbitrary shapes

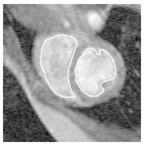




In all cases, we knew the explicit model to fit.

Today

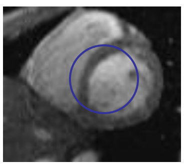
- Fitting an arbitrary shape with "active" deformable contours




Deformable contours

a.k.a. active contours, snakes

Given: initial contour (model) near desired object



[Snakes: Active contour models, Kass, Witkin, & Terzopoulos, ICCV1987]

Figure credit: Yuri Boykov

Deformable contours

a.k.a. active contours, snakes

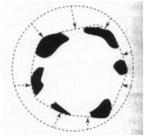
Given: initial contour (model) near desired object
Goal: evolve the contour to fit exact object boundary

Main idea: elastic band is iteratively adjusted so as to
- be near image positions with high gradients, **and**
- satisfy shape "preferences" or contour priors

[Snakes: Active contour models, Kass, Witkin, & Terzopoulos, ICCV1987]

Figure credit: Yuri Boykov

Deformable contours: intuition



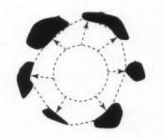
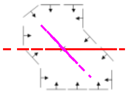


Image from http://www.healthline.com/blogs/exercise_fitness/uploaded_images/HandBand2-795868.JPG

Deformable contours vs. Hough

Like generalized Hough transform, useful for shape fitting; but



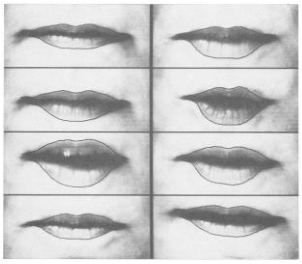
<u>Hough</u>
Rigid model shape
Single voting pass can detect multiple instances

<u>Deformable contours</u>
Prior on shape types, but shape iteratively adjusted (*deforms*)
Requires initialization nearby
One optimization "pass" to fit a single contour

Why do we want to fit deformable shapes?

- Some objects have similar basic form but some variety in the contour shape.

Why do we want to fit deformable shapes?



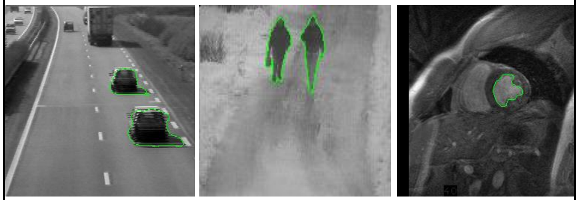
- Non-rigid, deformable objects can change their shape over time, e.g. lips, hands…

Figure from Kass et al. 1987

Why do we want to fit deformable shapes?

- Non-rigid, deformable objects can change their shape over time, e.g. lips, hands…

Why do we want to fit deformable shapes?



- Non-rigid, deformable objects can change their shape over time.

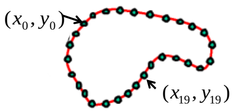
Figure credit: Julien Jomier

Aspects we need to consider

- Representation of the contours
- Defining the energy functions
  - External
  - Internal
- Minimizing the energy function
- Extensions:
  - Tracking
  - Interactive segmentation

Representation

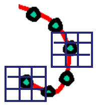
- We'll consider a discrete representation of the contour, consisting of a list of 2d point positions ("vertices").



$v_i = (x_i, y_i),$

for $i = 0, 1, \ldots, n-1$

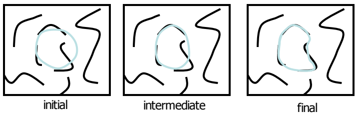
- At each iteration, we'll have the option to move each vertex to another nearby location ("state").



Fitting deformable contours

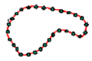
How should we adjust the current contour to form the new contour at each iteration?

- Define a cost function ("energy" function) that says how good a candidate configuration is.
- Seek next configuration that minimizes that cost function.



Energy function

The total energy (cost) of the current snake is defined as:



$$E_{total} = E_{internal} + E_{external}$$

Internal energy: encourage *prior* shape preferences: e.g., smoothness, elasticity, particular known shape.

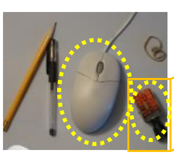
External energy ("image" energy): encourage contour to fit on places where image structures exist, e.g., edges.

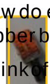
A good fit between the current deformable contour and the target shape in the image will yield a **low** value for this cost function.

External energy: intuition

- Measure how well the curve matches the image data
- "Attract" the curve toward different image features
  - Edges, lines, texture gradient, etc.

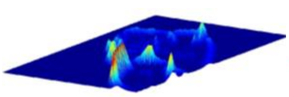
External image energy





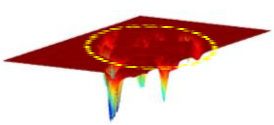
How do edges affect "snap" of rubber band?

Think of external energy from image as gravitational pull towards areas of high contrast



Magnitude of gradient

$$G_x(I)^2 + G_y(I)^2$$

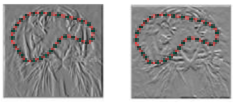


- (Magnitude of gradient)

$$-\left(G_x(I)^2 + G_y(I)^2\right)$$

External image energy

- Gradient images $G_x(x,y)$ and $G_y(x,y)$



- External energy at a point on the curve is:

$$E_{external}(\nu) = -(|G_x(\nu)|^2 + |G_y(\nu)|^2)$$

- External energy for the whole curve:

$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i,y_i)|^2 + |G_y(x_i,y_i)|^2$$

Internal energy: intuition

What are the underlying boundaries in this fragmented edge image?

And in this one?

Internal energy: intuition

A priori, we want to favor **smooth** shapes, contours with **low curvature**, contours similar to a **known shape**, etc. to balance what is actually observed (i.e., in the gradient image).

Internal energy

For a *continuous* curve, a common internal energy term is the "bending energy".

At some point *v(s)* on the curve, this is:

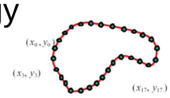
$$E_{internal}(\nu(s)) = \boxed{\alpha\left|\frac{d\nu}{ds}\right|^2} + \boxed{\beta\left|\frac{d^2\nu}{d^2s}\right|^2}$$

Tension, Elasticity

Stiffness, Curvature

Internal energy

- For our discrete representation,



$$v_i = (x_i, y_i) \qquad i = 0 \;....\; n-1$$

$$\frac{d\nu}{ds} \approx v_{i+1} - v_i \qquad \frac{d^2\nu}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

Note these are derivatives relative to ***position***---*not spatial image gradients.*

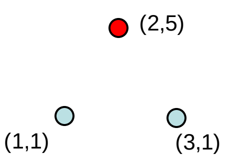
- Internal energy for the whole curve:

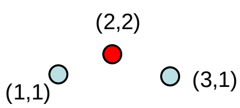
$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2 + \beta \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$

Why do these reflect ***tension*** *and* ***curvature****?*

Example: compare curvature

$$E_{curvature}(v_i) = \left\| v_{i+1} - 2v_i + v_{i-1} \right\|^2$$
$$= (x_{i+1} - 2x_i + x_{i-1})^2 + (y_{i+1} - 2y_i + y_{i-1})^2$$





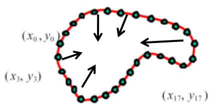
$$(3-2(2)+1)^2 + (1-2(5)+1)^2$$
$$= (-8)^2 = 64$$

$$(3-2(2)+1)^2 + (1-2(2)+1)^2$$
$$= (-2)^2 = 4$$

Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

$$E_{elastic} = \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2$$
$$= \alpha \cdot \sum_{i=0}^{n-1} (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2$$



What is the possible problem with this definition?

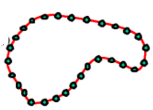
Penalizing elasticity

- Current elastic energy definition uses a discrete estimate of the derivative:

$$E_{elastic} \cancel{= \sum_{i=0}^{n-1} \alpha \left\| v_{i+1} - v_i \right\|^2}$$

Instead:

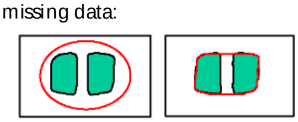
$$= \alpha \cdot \sum_{i=0}^{n-1} \left((x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 - \bar{d} \right)^2$$

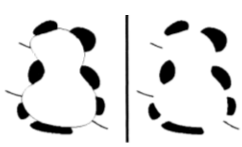


where d is the average distance between pairs of points – updated at each iteration.

Dealing with missing data

- The preferences for low-curvature, smoothness help deal with missing data:



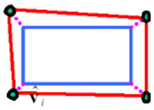


Illusory contours found!

[Figure from Kass et al. 1987]

Extending the internal energy: capture shape prior

- If object is some smooth variation on a known shape, we can use a term that will penalize deviation from that shape:



$$E_{internal} \mathrel{+}= \alpha \cdot \sum_{i=0}^{n-1} (v_i - \hat{v}_i)^2$$

where $\{\hat{v}_i\}$ are the points of the known shape.

Fig from Y. Boykov

Total energy: function of the weights

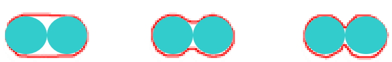
$$E_{total} = E_{internal} + \gamma E_{external}$$

$$E_{external} = -\sum_{i=0}^{n-1} |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

$$E_{internal} = \sum_{i=0}^{n-1} \alpha \left(\bar{d} - \|v_{i+1} - v_i\|\right)^2 + \beta \|v_{i+1} - 2v_i + v_{i-1}\|^2$$

Total energy: function of the weights

- e.g., α weight controls the penalty for internal elasticity

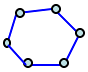


large α medium α small α

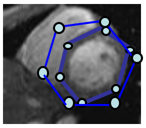
Fig from Y. Boykov

Recap: deformable contour

- A simple elastic snake is defined by:
  - A set of *n* points,
  - An internal energy term (tension, bending, plus optional shape prior)
  - An external energy term (gradient-based)



- To use to segment an object:
  - Initialize in the vicinity of the object
  - Modify the points to minimize the total energy

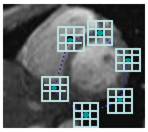


Energy minimization

- Several algorithms have been proposed to fit deformable contours.
- We'll look at two:
  - Greedy search
  - Dynamic programming (for 2d snakes)

Energy minimization: greedy

- For each point, search window around it and move to where energy function is minimal
  - Typical window size, e.g., 5 x 5 pixels



- Stop when predefined number of points have not changed in last iteration, or after max number of iterations
- Note:
  - Convergence not guaranteed
  - Need decent initialization

Energy minimization

- Several algorithms have been proposed to fit deformable contours.
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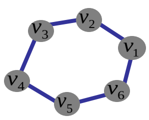
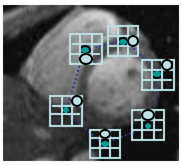
  - Greedy search
  - Dynamic programming (for 2d snakes)

Energy minimization: dynamic programming





With this form of the energy function, we can minimize using dynamic programming, with the *Viterbi* algorithm.

Iterate until optimal position for each point is the center of the box, i.e., the snake is optimal in the local search space constrained by boxes.

Fig from Y. Boykov
[Amini, Weymouth, Jain, 1990]

Energy minimization: dynamic programming

- Possible because snake energy can be rewritten as a sum of pair-wise interaction potentials:

$$E_{total}(v_1,\ldots,v_n) = \sum_{i=1}^{n-1} E_i(v_i, v_{i+1})$$

- Or sum of triple-interaction potentials.

$$E_{total}(v_1,\ldots,v_n) = \sum_{i=1}^{n-1} E_i(v_{i-1}, v_i, v_{i+1})$$

Snake energy: pair-wise interactions

$$E_{total}(x_1,\dots,x_n,y_1,\dots,y_n) \;=\; -\sum_{i=1}^{n-1} |G_x(x_i,y_i)|^2 + |G_y(x_i,y_i)|^2$$
$$+\;\alpha\cdot\sum_{i=1}^{n-1}(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2$$

Re-writing the above with $v_i = (x_i, y_i)$ *:*

$$E_{total}(v_1,\dots,v_n) \;=\; -\sum_{i=1}^{n-1}\|G(v_i)\|^2 \;+\; \alpha\cdot\sum_{i=1}^{n-1}\|v_{i+1}-v_i\|^2$$

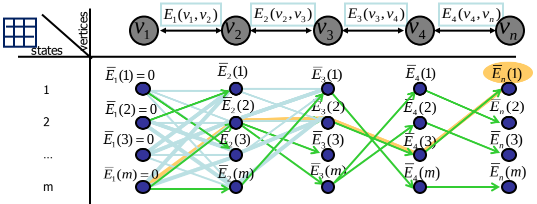
$$E_{total}(v_1,\dots,v_n) \;= E_1(v_1,v_2)+E_2(v_2,v_3)+\dots+E_{n-1}(v_{n-1},v_n)$$

where $E_i(v_i,v_{i+1}) = -\|G(v_i)\|^2 + \alpha\|v_{i+1}-v_i\|^2$

Viterbi algorithm

Main idea: determine optimal position (state) of predecessor, for each possible position of self. Then backtrack from best state for last vertex.

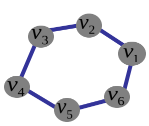
$$E_{total} = E_1(v_1,v_2)+E_2(v_2,v_3)+\dots+E_{n-1}(v_{n-1},v_n)$$

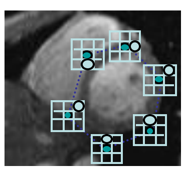


Complexity: $O(nm^2)$ vs. brute force search ____?

Example adapted from Y Boykov

Energy minimization: dynamic programming





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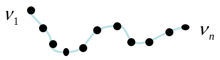
Iterate until optimal position for each point is the center of the box, i.e., the snake is optimal in the local search space constrained by boxes.

Fig from Y. Boykov
[Amini, Weymouth, Jain, 1990]

Energy minimization: dynamic programming

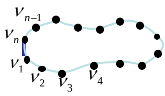
DP can be applied to optimize an open ended snake

$$E_1(v_1, v_2) + E_2(v_2, v_3) + ... + E_{n-1}(v_{n-1}, v_n)$$



For a closed snake, a "loop" is introduced into the total energy

$$E_1(v_1, v_2) + E_2(v_2, v_3) + ... + E_{n-1}(v_{n-1}, v_n) + \boxed{E_n(v_n, v_1)}$$



Work around:

1) Fix v_1 and solve for rest .
2) Fix an intermediate node at its position found in (1), solve for rest.

Aspects we need to consider

- Representation of the contours
- Defining the energy functions
  - External
  - Internal
- Minimizing the energy function
- Extensions:
  - Tracking
  - Interactive segmentation

Tracking via deformable contours

1. Use final contour/model extracted at frame *t* as an initial solution for frame *t+1*
2. Evolve initial contour to fit exact object boundary at frame *t+1*
3. Repeat, initializing with most recent frame.

Tracking Heart Ventricles (multiple frames)

Tracking via deformable contours


Visual Dynamics Group, Dept. Engineering Science, University of Oxford.

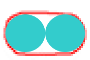
Applications: Traffic monitoring
Human-computer interaction
Animation
Surveillance
Computer assisted diagnosis in medical imaging

3D active contours

Jörgen Ahlberg
http://www.cvl.isy.liu.se/ScOut/Masters/Papers/Ex1708.pdf

Limitations

- May over-smooth the boundary



- Cannot follow topological changes of objects

Limitations

- External energy: snake does not really "see" object boundaries in the image unless it gets very close to it.

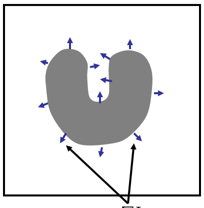
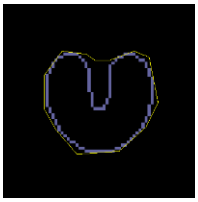


image gradients ∇I are large only directly on the boundary

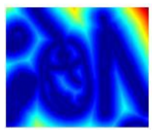


Distance transform

- External image can instead be taken from the **distance transform** of the edge image.

original

-gradient



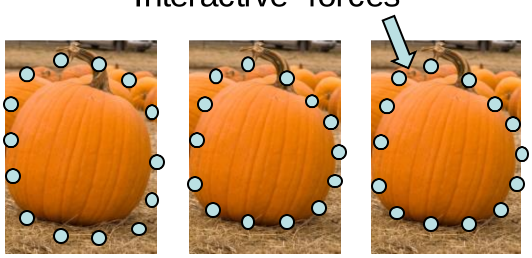
distance transform

edges

Value at (x,y) tells how far that position is from the nearest edge point (or other binary mage structure)

```
>> help bwdist
```

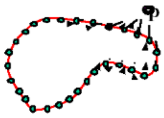
Interactive forces



How can we implement such an *interactive* force with deformable contours?

Interactive forces

- An energy function can be altered online based on user input – use the cursor to push or pull the initial snake away from a point.
- Modify external energy term to include:



$$E_{push} = \sum_{i=0}^{n-1} \frac{r^2}{|v_i - p|^2}$$

Nearby points get pushed hardest

Intelligent scissors

Another form of interactive segmentation:

Compute optimal paths **from every point to the seed** based on edge-related costs.

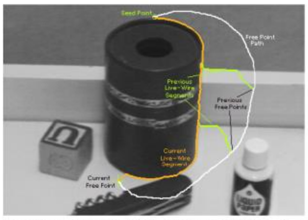
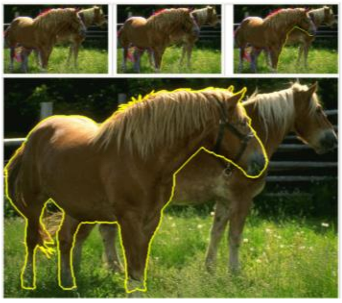


Figure 2: *Image demonstrating how the live-wire segment adapts and snaps to an object boundary as the free point moves (via cursor movement). The path of the free point is shown in white. Live-wire segments from previous free point positions (t_0, t_1, and t_2) are shown in green.*

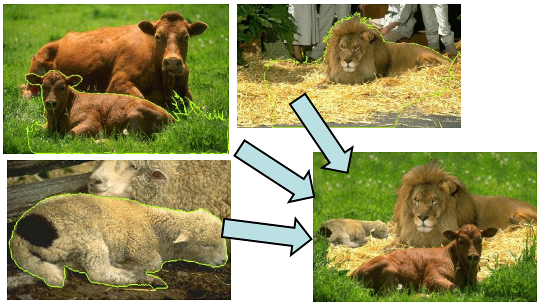
[Mortensen & Barrett, SIGGRAPH 1995, CVPR 1999]

Intelligent scissors



- http://rivit.cs.byu.edu/Eric/Eric.html

Intelligent scissors



- http://rivit.cs.byu.edu/Eric/Eric.html

Deformable contours: pros and cons

Pros:

- Useful to track and fit non-rigid shapes
- Contour remains connected
- Possible to fill in "subjective" contours
- Flexibility in how energy function is defined, weighted.

Cons:

- Must have decent initialization near true boundary, may get stuck in local minimum
- Parameters of energy function must be set well based on prior information

Summary

- Deformable shapes and active contours are useful for
  - Segmentation: fit or "snap" to boundary in image
  - Tracking: previous frame's estimate serves to initialize the next
- Fitting active contours:
  - Define terms to encourage certain shapes, smoothness, low curvature, push/pulls, …
  - Use weights to control relative influence of each component cost
  - Can optimize 2d snakes with Viterbi algorithm.
- Image structure (esp. gradients) can act as attraction force for *interactive* segmentation methods.